Soundly managing uncertain decisions in diagnostic analysis

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Abstract. This paper presents two diagnostic methods, which are able to handle large scale distributed industrial plants. It presents logical approaches providing diagnoses in presence of doubts in detection test results. Two fuzzy logical based approaches are proposed in order to soundly take into account doubts in the decisions provided by detection tests. The first one relies on a structural analysis introduced by the FDI community and the second one relies on logical analysis based on the IA approach of diagnosis. These techniques have been implemented in the EC project named MAGIC.

Keywords. Consistency Based Diagnosis, Signature Table Based Diagnosis, Fault Detection and Isolation, Multi-agent System, Uncertain detection decision, Fuzzy logic, Complex systems.

1 Introduction

Monitoring and diagnosis of physical systems is a major research area in control and computer sciences. Many different kinds of detection algorithms dedicated to dynamical systems have been designed by control scientists (Frank *et al.*, 2000; Isermann, 1997; Patton et al, 1997). The probably most common approaches rely on signal processing and on analytical models representing the system's behavior to be diagnosed. These models are used to design detection tests generating symptoms usually either based on parity relations or on state observers.

Most of the detection algorithms are useful depending on the information available about systems to be monitored. Therefore, common issues of industrial applications are: firstly, they require different kinds of detection algorithms and, secondly, physical systems are spatially distributed.

This particular problem is tackled by an EC-research project called, "Multi-Agents-based Diagnostic Data Acquisition and Management in Complex Systems (MAGIC)". It aims at providing general purpose distributed software architecture and a set of tools that may be used for detection and diagnosis analysis of complex dynamical systems (Köppen-Seliger et al., 2002). As mentioned in the project name, the MAGIC system relies on a multi-agent paradigm. Each agent of the MAGIC system is specialized. Each detection algorithm within this architecture is embedded in an agent able to communicate with the rest of the MAGIC system Agents. The diagnostic analysis, which is in the scope of this paper, is handled by another specialized agent called "Diagnostic Decision Agent". This agent has to cope with the problem of computing diagnoses whereas it does not have much information about each detection algorithm. This paper describes the information that each detection algorithm is supposed to provide to this agent and it also presents how this information is processed in order to compute the possible diagnoses.

Diagnosis is usually decomposed into two steps. The detection step provides symptoms from detection tests relying, for instance, on analytical, signal or data based models. The symptoms result usually from a decision based on thresholds. It determines whether there are faults in the sub-system checked or not. However, the dichotomic nature of the decisions added to the rigorous meaning of the inconsistent result (Ploix et al, 2003a) leads to increase the detection thresholds in order to avoid false alarm. Therefore, an insensitivity of the diagnostic system is resulting. To improve the diagnostic system performances, MAGIC system allows symptoms in the form of degrees of truth i.e. uncertain decisions providing that the uncertainty is quantified. It is presented in section 3.

The isolation step collects symptoms from the detection one. It has to provide all the possible diagnoses. A diagnosis may be literally defined as a state of a physical system consistent with the available symptoms. Two different kinds of diagnostic analysis have been developed. FDI community has developed signature table and directional residuals based approached (Franck, 1996) whereas AI community has developed logical approaches (Reiter, 1987; De Kleer, 1987; Greiner, 1989; Poole, 1988). Nevertheless, there are some links between the logical approach and the control ones, which have been studied recently (Cordier, *et al.*, 2000) (Ploix et al, 2003b). Section 4 examines how structural diagnostic analysis provided by the FDI community may be adapted in order to take into account uncertainty in decision. Section 5 examines how logical diagnostic analysis may take into account uncertainties in detection test decisions.

The methods proposed are direct extensions of crisp logic reasoning to fuzzy logic and avoid the introduction of necessity measure as suggested in (Cayrac et al, 1996).

2 Problem Statement

As detailed in (Ploix et al, 2003 a), some information about detection tests is required for diagnostic analysis to process diagnoses. Let's first define a detection test and then point out the required information. Firstly, a detection test implicitly refers to physical variables representing measurable model-independent knowledge about actual system states. The notation [x] means that x is a physical variable (or a vector of physical variables). These variables may lead to known values or not. $\tilde{x}$ denotes the known values of [x]. It may be provided by measurement or it may be known because a variable is controlled or because its value is assumed (an ambient temperature may be assumed to be equal to 20°c).

For fault diagnosis, a component, denoted by C_j may be described by one or several elementary models:

$$EM_i: \left(r_i\left([x],\tilde{x}\right), v_i\left([x],\tilde{x}\right), h_i\left(C_j\right)\right) \quad (1)$$

where

- $r_i\left([x],\tilde{x}\right)$ denotes an elementary analytical relation, called constraint in (Declerck et al, 2000), representing partially or totally the behaviour of a component C_i in a given state h_i. It is a relation between physical variables and, possibly, known values. It may be any kinds of model such as differential equations, neural networks,...
- $v_i\left([x],\tilde{x}\right)$ defines the validity of the previous relation, in other words, it depicts when the elementary analytical relation is suitable or not.
 h_i is a logical proposal, called component state, of which the subject is a component C_j and the predicate is a state. This proposal depicted the component state modelled by the elementary relation and the validity of this elementary model. Notation coming from circumscription theory (Mc Carthy, 1986) has been adopted: a component C_j behaving normally will be described by $h_i = \neg AN\left(C_j\right)$. If this component is altered by a specific fault D, it will be written as follow: $h_i = D\left(C_j\right)$. Note that the following implication always holds: $D\left(C_j\right) \Rightarrow AN\left(C_j\right)$ but the equivalence would be false.

A subsystem model is a set of elementary models EM_i (in order to simplify the notations, the variables intervening in elementary models are omitted). Its behaviour is depicted by the union of all the involved elementary models relations. Its validity is equal to the conjunction of elementary model's validities. The state of the subset is equal to the conjunction of the component states of the elementary models. Nevertheless, a subsystem exists only if there is no contradictions in component states (gathering $\neg AN\left(C_j\right)$ and $fault\left(C_j\right)$ in the same subsystem for example) and if there at least a valid set point (avoiding the validities $\tilde{h} > 50$ and $\tilde{h} < 30$ in the same subsystem for example). It means that there must be some operating set points where the subsystem model is valid and where elementary model states do not contain two contradictory states:

$$\left(\bigcup_{i\in\phi} r_i\left([x],\tilde{x}\right), \bigwedge_{i\in\phi} v_i\left([x],\tilde{x}\right), \bigwedge_{i\in\phi} h_i\right) \quad (1)$$

$$\text{with } \bigwedge_{i\in\phi} h_i \not\equiv false \text{ and } \exists\tilde{x} / \bigwedge_{i\in\phi} v_i\left(\tilde{x}\right) = true$$

A subsystem will be qualified as testable when it may lead to a detection test, which is a testing function containing only known variables. Methods for discovering testable subsystem, sometimes called Analytical Redundancy Relation, are proposed in (Declerck et al, 1991).

Because detection tests are distributed, only some of their characteristics are known by the diagnostic analysis system. Let's define a detection test as it is known by the diagnostic analysis system.

Let Σ be a physical system made up of a set $\mathcal{C}_\Sigma$ of components, whose individual elements are denoted by C_i. Let $\mathcal{H}_\Sigma$ be the set of modelled component states h_i of Σ:

$$\mathcal{H}_\Sigma \left\{h_j : \mathcal{C}_\Sigma \rightarrow \left\{true,\ false\right\}\right\} \quad (2)$$

Let $\tau(t)$ be a temporal sliding sequence representing the sampling times for known values required at each detection algorithm step:

$$\mathcal{T} : \left\{\tau : \mathbb{R} \rightarrow \mathbb{R}^r / \tau(t) = \left\{t+\Delta_1, \ldots, t+\Delta_r\right\}; \forall i, \Delta_i \in \mathbb{R}\right\} \quad (3)$$

Let $\tilde{\mathcal{X}}$ be a set of observation sequences related to the time sequence (3):

$$\tilde{\mathcal{X}}_\Sigma : \left\{\tilde{x}\left(\tau(t)\right) \in \mathbb{R}^n : n \in \mathbb{N}, t \in \mathbb{R}\right\} \quad (4)$$

Then, a function t_k, defined by:

$$t_k : \begin{cases} \mathcal{H}_{t_k} \subset \mathcal{H}_\Sigma \\ x_{t_k} \subset \tilde{\mathcal{X}}_{t_k}\left(\tau(t)\right) \\ \neg\sigma_{t_k}\left(x_{t_k}\right) \\ \upsilon_{t_k}\left(x_{t_k}\right) \end{cases} \quad (5)$$

where $\neg\sigma_{t_k}\left(x_{t_k}\right)$ and $\upsilon_{t_k}\left(x_{t_k}\right)$ are logical proposal

is a detection test if:

$$\forall\tilde{x}_{t_k} / \neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right) \wedge \upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \Rightarrow \neg\hat{\mathcal{H}}_{t_k} \text{ where } \hat{\mathcal{H}}_{t_k} = \bigwedge_{h_i \in \mathcal{H}_{t_k}} h_i \quad (6a)$$

$$\forall\tilde{x}_{t_k} / \neg\upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \Rightarrow \text{ no information about } \hat{\mathcal{H}}_{t_k} \quad (6b)$$

where $\neg\sigma_{t_k}(.)$ is a testing function and υ_{t_k} is a validity function.

Relation (6a) is the basic definition on a detection test. Because of the implication, it does not assume exoneration. Relation (6b) means that if a detection test is invalid, the detection test related component states may be either true or false.

Because the detection test may lead to uncertain decision, fuzzy logic concepts have been used (Zadeh, 1965). The truths of testing and validity functions $\neg\sigma_{t_k}$ and υ_{t_k} may belong to [0 1]. $\neg\sigma_{t_k} = 1$ means that the detection test is surely inconsistent and 0 that it is surely consistent. 0.5 stands for dubious decision (impossibility to decide between consistent and inconsistent decision). For instance, 0.2 means that the possibility for inconsistency is 20%, and, in a complementary way, the possibility of being consistent is 80%. These symptoms are provided by the distributed detection tests.

The truth of validity is also a membership degree. It depicts for instance whether the related model is valid at the current set point or not. Consider for instance a linear model designed to fit around the vicinity of the operating set point. The validity will tend to zero if the actual set point move far from the reference one.

3 Symptom processing

Symptoms, composed by rough results $\neg\sigma_{t_k}(.)$ and υ_{t_k}, should be collected from distributed detection tests. However, these rough results are not directly used for diagnostic analysis. Firstly, for each detection test, decision $\neg\sigma_{t_k}$ and validity υ_{t_k} have to be merged in order to get the truth, denoted by *T*(.), of the detection test related hypothesis $\hat{H}_{t_k}$ defined in (5) and (6a). Then, the past results have to be considered in the diagnostic analysis.

Merging the results $\neg\sigma_{t_k}(.)$ and υ_{t_k} has to be done according to detection test definitions (6a) and (6b).

First consider (6a). It may be transformed according to the following inferences:

$$\begin{array}{l} \rightarrow \neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right) \wedge \upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \Rightarrow \neg\hat{\mathcal{H}}_{t_k} \\ \hline \rightarrow \neg\left(\neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right) \wedge \upsilon\left(\tilde{x}_{t_k}\right)\right) \vee \neg\hat{\mathcal{H}}_{t_k} \\ \hline \rightarrow \neg\left(\neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right)\right) \vee \neg\upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \vee \neg\hat{\mathcal{H}}_{t_k} \end{array} \tag{7}$$

The truth of the last proposal may be calculated according to fuzzy logic arithmetic. Summation has been used for disjunction:

$$\begin{array}{l} \rightarrow T\left(\neg\left(\neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right)\right) \vee \neg\upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \vee \neg\hat{\mathcal{H}}_{t_k}\right) = 1 \\ \hline \rightarrow \min\left(1, \left(1-\mu_{\neg\sigma}\right) + \left(1-\mu_{\upsilon}\right) + \mu_{\neg\hat{\mathcal{H}}}\right) = 1 \\ \hline \rightarrow \min\left(1, 2-\mu_{\neg\sigma}-\mu_{\upsilon}+\mu_{\neg\hat{\mathcal{H}}}\right) = 1 \\ \hline \rightarrow \mu_{\neg\hat{\mathcal{H}}} \geq \mu_{\neg\sigma}+\mu_{\upsilon}-1 \end{array} \tag{8}$$

where

$$\mu_{\neg\sigma} = T\left(\neg\sigma_{t_k}\right),\ \mu_{\upsilon} = T\left(\upsilon_{t_k}\right) \text{ and } \mu_{\neg\hat{\mathcal{H}}} = T\left(\neg\hat{\mathcal{H}}_{t_k}\right)$$

Consider now the definition (6b). It leads to the following property:

$$\mu_{\upsilon} = 0 \Rightarrow \mu_{\neg\hat{\mathcal{H}}} = 0.5 \tag{9}$$

Therefore, a merging function has to satisfy the properties (8) and (9). Nevertheless, lots of functions may be possible. The function should hold for any kinds of diagnostic analysis, including structural analysis coming from FDI community, which exonerates assumptions of detection tests without alarms. A merging function still valid in case of exoneration would be more interesting. However, the existence of such a merging function has to be checked. If it exists, then the following proposal should thus be considered as true:

$$\begin{array}{l} \rightarrow \forall\tilde{x}_{t_k} / \sigma_{t_k}\left(\tilde{x}_{t_k}\right) \wedge \upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \Rightarrow \hat{\mathcal{H}}_{t_k} \\ \hline \rightarrow \forall\tilde{x}_{t_k} / \neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right) \vee \neg\upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \vee \hat{\mathcal{H}}_{t_k} \end{array}$$

The truth of this proposal may be written as follows:

$$\begin{array}{l} \rightarrow T\left(\neg\sigma_{t_k}\left(\tilde{x}_{t_k}\right) \vee \neg\upsilon_{t_k}\left(\tilde{x}_{t_k}\right) \vee \hat{\mathcal{H}}_{t_k}\right) = 1 \\ \hline \rightarrow \min\left(1, \mu_{\neg\sigma} + \left(1-\mu_{\upsilon}\right) + \left(1-\mu_{\neg\hat{\mathcal{H}}}\right)\right) = 1 \\ \hline \rightarrow \min\left(1, 2+\mu_{\neg\sigma}-\mu_{\upsilon}-\mu_{\neg\hat{\mathcal{H}}}\right) = 1 \\ \hline \rightarrow \mu_{\neg\hat{\mathcal{H}}} \leq 1+\mu_{\neg\sigma}-\mu_{\upsilon} \end{array} \tag{10}$$

The possible area for a merging function satisfying properties (8), (9) and (10) have been draw in figure 1: it corresponds to the hatched area. Therefore, a unique merging function valid for diagnostic analyzes with or also without exoneration exists.

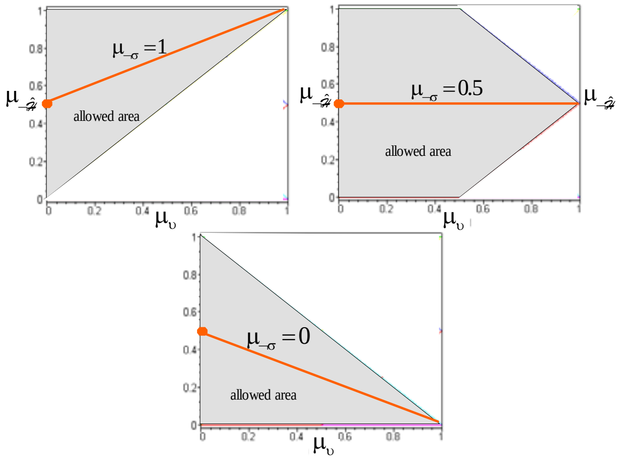

Figure 1 - Possible area for a merging function

The unique linear merging function has been drawn in figure 1. It may be written:

$$\mu_{\neg\hat{\mathcal{H}}} = \frac{1+\left(2\mu_{\neg\sigma}-1\right)\mu_{\upsilon}}{2} \tag{11}$$

$\mu_{\neg\sigma}$ may be seen as a membership to the decision of inconsistency between measurement and model i.e. the membership to alarm decision. $\mu_{\neg\sigma} = 1$ means that there is an inconsistency without any possible doubt. $\mu_{\neg\sigma} = 0$ means that there is no reason for suspecting an inconsistency and $\mu_{\neg\sigma} = 0.5$ means that it is impossible to decided between alarm or not. These decisions may be obtained thanks to 2 thresholds on residuals as shown in figure 2.

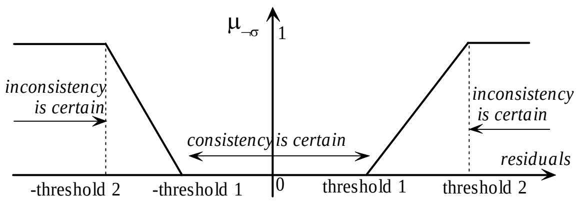

Figure 2 – A way of making uncertain decisions

The merging function shows that the confidence level $\mu_{\neg\hat{\mathcal{H}}}$ is equal to the decision $\mu_{\neg\sigma}$ if the validity is equal to 1. The confidence level regularly tends to 0.5 (impossible to decide) when the validity of the detection test decreases.

Confidence levels related to a detection test are performed at different times. However, decisions are not independent. For instance, if $\mu_{\neg\hat{\mathcal{H}}}(t)=1$ and if $\mu_{\neg\hat{\mathcal{H}}}(t')=0$ when *t'*>*t*, it means that an inconsistency has been detected at time *t* and that it has disappeared at time *t'*. However, a fault does not necessary lead to a alarm at any time: a fault may be visible only during some operating modes or during some transient behaviours. Therefore, even if $\mu_{\neg\hat{\mathcal{H}}}(t')=0$, the final decision, denoted by $\mu^*_{\neg\hat{\mathcal{H}}}(t')$, may infer that an inconsistency has been detected and that it is still considered. A forgetting sliding time window has to be defined.

If all the components are subject to permanent fault, i.e. the component states cannot change without a human operation, the size of forgetting sliding time window will be infinite and the worst case, i.e. the best symptom indication, has to be memorized:

$$\mu^*_{\neg\hat{\mathcal{H}}}(t) = \sup_{0\le\tau\le t}\left(\mu_{\neg\hat{\mathcal{H}}}(\tau)\right)$$

Nevertheless, all components may not be subject to permanent faults, in particular in electronics and computer systems. Consequently, each component C_i is characterized by a typical recovery time $\tau(C_i)$, which represents the time period below of which the component C_i can not have recovered its normal state.

Considering a detection test t_k, the typical recovery time corresponds to the time where normal state may be recovered whatever the abnormal state was. Because the abnormal state is unknown, the size of forgetting sliding time window corresponds to the minimum of the typical recovery time of the involved components:

$$\tau_r = \min_{h_i\in\mathcal{H}_{t_i}} \tau\left(\mathcal{C}(h_i)\right)$$

The final confidence level is given by:

$$\mu^*_{\neg\hat{\mathcal{H}}}(t) = \sup_{0\le\tau\le \min_{h_i\in\mathcal{H}_{t_i}}\tau(\mathcal{C}(h_i))}\left(\mu_{\neg\hat{\mathcal{H}}}(\tau)\right) \qquad (12)$$

For the sake of simplicity, no distinction will be made in the following between $\mu^*_{\neg\hat{\mathcal{H}}}(t)$ and $\mu_{\neg\hat{\mathcal{H}}}(t)$.

4 Managing uncertain decision in structural diagnostic analysis

This section examines diagnostic analysis method usually performed by the FDI community. This approach is named structural approach. It consists in directly comparing alarms with the columns of the fault signature table.

As an example, table 1 summarizes three detection tests t_1, t_2 and t_3. Each test t_i is related to hypotheses h_j from $\mathcal{H}_{ti}$, i.e. to possible faults. For instance, the test t_1 checks whether components C_1, C_3 and C_4 are normal. Therefore, test t_1 is sensitive to a fault on C_1 but insensitive to a fault on C_2.

$\neg AN(.)$	C_1	C_2	C_3	C_4
t_1	1	0	1	1
t_2	0	1	1	0
t_3	1	0	1	0

Table 1 - Example of fault signature table

In a crisp logic framework, $\neg\mathcal{H}_{t1}$, $\mathcal{H}_{t2}$ and $\neg\mathcal{H}_{t3}$ will lead to the conclusion that $AN(C_1)$ is a diagnosis because its fault signature is consistent with the symptoms. The issue is now to look for fault signatures when detection tests lead to uncertain decisions.

Let $[\lambda_{ij}]$ be a matrix representing a fault signature table such as table 1 where i refers to the i^{th} detection test and j to the j^{th} assumption h_j. The fault signature table is defined by:

$$\begin{cases} h_j \in \mathcal{H}(t_i) \Leftrightarrow \lambda_{ij} = 1 \\ h_j \notin \mathcal{H}(t_i) \Leftrightarrow \lambda_{ij} = 0 \end{cases} \qquad (13)$$

Logically speaking, a fault signature will be consistent with the detection test result if:

$$\forall t_i, \left(h_j \in \mathcal{H}(t_i)\right) \Leftrightarrow \neg\hat{\mathcal{H}}(t_i) \qquad (14)$$

If (14) is satisfied, then $\neg h_j$ is a diagnosis. The truth of (14) may be reformulated:

$$\frac{\dfrac{\to \forall t_i, \left(h_j \in \mathcal{H}(t_i)\right) \Leftrightarrow \neg\hat{\mathcal{H}}(t_j)}{\to \bigwedge_{t_i}\left(\left(h_j \in \mathcal{H}(t_i)\right) \Leftrightarrow \neg\hat{\mathcal{H}}(t_i)\right)}}{\to \bigwedge_{t_i}\left(\left(\left(h_j \in \mathcal{H}(t_i)\right)\wedge\neg\hat{\mathcal{H}}(t_i)\right)\vee\left(\left(h_j \notin \mathcal{H}(t_i)\right)\wedge\hat{\mathcal{H}}(t_i)\right)\right)}$$

The truth of the last expression may be calculated according to the fuzzy logic arithmetic:

$$\begin{aligned}\mu_{\neg h_j} &= T\left(\bigwedge_{t_i}\begin{pmatrix}\left(\left(h_j \in \mathcal{H}(t_i)\right)\wedge\neg\hat{\mathcal{H}}(t_i)\right)\vee\ldots \\ \left(\left(h_j \notin \mathcal{H}(t_i)\right)\wedge\hat{\mathcal{H}}(t_i)\right)\end{pmatrix}\right) \\ &= \min_{t_i}\left(\lambda_{ij}\mu_{\neg\hat{\mathcal{H}}(t_i)} + (1-\lambda_{ij})(1-\mu_{\neg\hat{\mathcal{H}}(t_i)})\right)\end{aligned} \qquad (15)$$

Operator min(.) has been chosen as T-norm and summation as T-conorm.

Assume that the detection tests depicted by table 1 lead to the following results:

$$\left(\mu_{\neg\hat{\mathcal{H}}(t_1)}, \mu_{\neg\hat{\mathcal{H}}(t_2)}, \mu_{\neg\hat{\mathcal{H}}(t_3)}\right) = (0.8, 0.5, 0.4) \qquad (16)$$

Table 2 summarizes the truths of the different diagnoses.

Diagnosis	$AN(C_4)$	$AN(C_1)$	$AN(C_3)$	$AN(C_2)$
Truth	0.50	0.40	0.40	0.20

Table 2 - Structural diagnoses

Diagnosis $AN(C_4)$ is the most plausible diagnosis. It is not really surprising because of the strong likeness between its signature and the collected symptoms.

5 Managing uncertain decision in logical analysis

Logical, also called consistency based, diagnostic analysis has been introduced in (Reiter, 1987) and in (de Kleer et al, 1987). This section deals with an extension of this theory to dynamical system provided in (Ploix et al, 2003a) able to cope with state observers and parity relations. It examines how uncertain decision influence logical analysis.

1.1 Diagnoses calculation

A distinction is made between situation where all the results provided by detection tests are not surely inconsistent and situation where there exists at least one surely inconsistent detection test.

The first situation is defined by: $\forall t_i, \mu_{\neg\hat{\mathcal{H}}_{t_i}} \in [0,1[$. In this case, the possible diagnoses are given by:

$$D_{\min} = \left\{ \neg h_j \,/\, h_j \in \bigcup_{t_i / \mu_{\neg \mathcal{H}_{t_i}} \in]0,1[} \mathcal{H}_{t_i} \right\} \quad (17)$$

Let's prove this result. Let $\mathcal{T}_u$ be the set of uncertain detection tests: $\mathcal{T}_u = \left\{ t_i / \mu_{\neg \hat{\mathcal{H}}_{t_i}} \in]0,1[\right\}$.

By definition, each test of $\mathcal{T}_u$ may lead to a consistent or an inconsistent result. Considering situation where only one test $t_i \in T_u$ is inconsistent, the minimal diagnoses are given by: $\left\{ \neg h_j / h_j \in \mathcal{H}(t_i) \right\}$.

Consider now situations where several tests $t_{i:i\in\blacklozenge}$ are inconsistent. Each diagnosis may contain one or several hypotheses $\neg h_j$. However each fault necessary appears at least in one of the diagnosis obtained in considering only one inconsistent test $t_{i:i2\phi}$. Because only minimal diagnoses are considered, only diagnoses containing only one faulty assumption has to be considered. All the minimal diagnoses are then given by:

$$\bigcup_{t_i \in \mathcal{T}_u} \left\{ \neg h_j / h_j \in \mathcal{H}(t_i) \right\}$$

This union yields (17).

Let's consider now the second situation where there exists at least one surely inconsistent detection test. Let $\mathcal{T}_c$ be the set of surely inconsistent detection tests:

Let $\mathcal{D}_c$ be the logical diagnoses calculated according to the Reiter's algorithm. Each element d_j of $\mathcal{D}_c$ explains the conflicts bring about inconsistencies involved in $\mathcal{T}_c$.

If detection tests from $\mathcal{T}_c$ cannot be consistent, other detection tests from $\mathcal{T}_u$ may lead either to consistent or to inconsistent results. If all the tests of $\mathcal{T}_u$ are consistent, no more conflict appears and the diagnoses will be given by $\mathcal{D}_c$. Assume now that some detection tests of $\mathcal{T}_u$ lead to inconsistent results. Therefore, a diagnosis d_j from $\mathcal{D}_c$ may not explain the additional conflicts and some more hypotheses $\neg h_j$ should be added to d_j in order to explain all the conflicts. However, because d_j is also a diagnosis, d_j plus additional assumption will not be minimal. Consequently, the minimal diagnoses are all given by $\mathcal{D}_c$.

1.2 Truth of the diagnoses

The two previous situations will be also distinguished in this part.

In the first situation where $\mathcal{T}_c$=1, the truth of a diagnosis $\neg h_j$ is equal by definition to:

$$T(\neg h) = 1 - T(h) \quad (18)$$

$T(h_j)$ may be calculated in searching the detection results such as $\neg h_j$ is not involved into any diagnosis.

Denoting $\mathcal{T}_{hj}$, the set of detection tests, which do not refer to h_j:

$$\mathcal{T}_{h_j} = \left(t_i / h_j \in \mathcal{H}(t_i) \right)$$

all the detection results that do not involved h_j are given by:

$$h_j = \bigwedge_{t_i \in \mathcal{T}_{h_j}} \hat{\mathcal{H}}(t_i)$$

Therefore, the truth of the expression (18) may be calculated:

$$\begin{aligned} &\to T(\neg h_j) = T\left(\neg \bigwedge_{t_i \in \mathcal{T}_{h_j}} \hat{\mathcal{H}}(t_i) \right) = T\left(\bigvee_{t_i \in \mathcal{T}_{h_j}} \neg \hat{\mathcal{H}}(t_i) \right) \\ &\to \mu_{\neg h_j} = \max_{t_i \in \mathcal{T}_{h_j}} \left(\mu_{\neg \hat{\mathcal{H}}(t_i)} \right) \end{aligned} \quad (19)$$

Let consider now the second situation where $T_c \neq 1$. The truth of each diagnosis d_j of $\mathcal{D}_c$ has to be calculated. The diagnosis d_i will appear if:

$$d_i = \bigwedge_{t_i \in \mathcal{T}_c} \neg \hat{\mathcal{H}}(t_i)$$

Because of the minimality, the diagnosis is independent of the results of the detection tests of $\mathcal{T}_u$. Therefore,

$$\forall d_j \in \mathcal{D}_c, T(d_j) = T\left(\bigwedge_{t_i \in \mathcal{T}_c} \neg \hat{\mathcal{H}}(t_i) \right) = 1 \quad (20)$$

1.3 Circumstantial plausibility

Logical diagnostic analysis do not consider consistent detection tests because there results are unreliable (a consistent result does not prove the absence fault). Nevertheless, consistent detection tests may provide some interesting information.

Therefore, a circumstantial plausibility (also called contextual measurement) has been introduced in (Ploix et al, 2003a). Here, a slightly different circumstantial plausibility has been introduced. It is rather closed to the structural approach because it checks if the fault signatures are closed to the detection results. However, because a strict equivalence between a fault signature and the detection test results is a strong constraint, a hamming distance between detection test results and the expected symptom of a fault is calculated. This distance is based on a 1-norm. It may be logically defined by:

$$\eta(d) = \frac{\sum_{t_i \in \mathcal{T}} T\left(\neg \exists h_i \in \left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \right) \Leftrightarrow \hat{\mathcal{H}}(t_i) \right)}{card(\mathcal{T})} \quad (21)$$

The main expression is transformed by logical inferences:

$$\begin{aligned} &\to \neg \exists h_i \in \left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \right) \Leftrightarrow \hat{\mathcal{H}}(t_i) \\ \hline &\to \left(\exists h_i \in \left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \right) \vee \hat{\mathcal{H}}(t_i) \right) \wedge \ldots \\ &\quad \ldots \left(\neg \exists h_i \in \left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \right) \vee \neg \hat{\mathcal{H}}(t_i) \right) \\ \hline &\to \neg \begin{pmatrix} \left(\neg \exists h_i \in \left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \right) \wedge \neg \hat{\mathcal{H}}(t_i) \right) \vee \ldots \\ \left(\exists h_i \in \left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \right) \wedge \hat{\mathcal{H}}(t_i) \right) \end{pmatrix} \end{aligned}$$

The truth of the last expression leads to:

$$1 - \begin{pmatrix} T\left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) = \varnothing \right) \mu_{\neg \hat{\mathcal{H}}(t_i)} + \ldots \\ T\left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \neq \varnothing \right) \left(1 - \mu_{\neg \hat{\mathcal{H}}(t_i)} \right) \end{pmatrix} = \ldots$$

$$\ldots = \begin{pmatrix} T\left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) = \varnothing \right) \left(1 - \mu_{\neg \hat{\mathcal{H}}(t_i)} \right) + \ldots \\ T\left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \neq \varnothing \right) \mu_{\neg \hat{\mathcal{H}}(t_i)} \end{pmatrix}$$

Thus, definition (21) leads to:

$$\eta(d) = \frac{\sum_{t_i \in \mathcal{T}} \left(\begin{array}{l} T\left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) = \varnothing\right)\left(1 - \mu_{\neg\hat{\mathcal{H}}(t_i)}\right) + \dots \\ \dots T\left(\mathcal{H}(\neg d) \cap \mathcal{H}(t_i) \neq \varnothing\right)\mu_{\neg\hat{\mathcal{H}}(t_i)} \end{array} \right)}{card(\mathcal{T})} \quad (22)$$

1.4 Example

Consider a set of detection tests depicted by the table 1 and the detection test results given in (16). Formal plausibility has been computed according to formula (19) and circumstantial plausibility according to formula (22).

Diagnosis	$AN(C_4)$	$AN(C_1)$	$AN(C_3)$	$AN(C_2)$
Formal	0.80	0.80	0.80	0.50
Circumstantial	0.63	0.57	0.57	0.43

Table 3 - Logical diagnoses

Suppose now that the detection test results are:

$$\left(\mu_{\neg\hat{\mathcal{H}}(t_1)}, \mu_{\neg\hat{\mathcal{H}}(t_2)}, \mu_{\neg\hat{\mathcal{H}}(t_3)}\right) = (1, 1, 0.3)$$

The logical results are summarized in table 4:

Diagnosis	$AN(C_3)$	$AN(C_2)$^ $AN(C_4)$	$AN(C_3)$
Formal	1	1	1
Circumstantial	0.77	0.77	0.77

Table 4 - Logical diagnoses

It points out that the results of the crisp logical diagnostic analysis may be found in uncertain situations.

6 Conclusion

This paper shows how to manage uncertain decisions of detection tests in diagnostic analysis of large scale distributed complex systems where very few information is known about each detection algorithm. The results are both extensions of structural analysis coming from FDI community and of logical analysis coming from DX community. They have been inferred by extension of crisp logic definitions to fuzzy logic definitions.

The results provided in this paper allow handling situations where fault are not fully confirmed. In such cases, indications on possibly faulty component or on deteriorating components may be provided.

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