

# FAULT DIAGNOSIS REASONING FOR SET-MEMBERSHIP APPROACHES AND APPLICATION

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**Abstract.** This paper focuses on diagnostic reasoning for uncertain dynamical systems. It proposes a diagnostic reasoning that distinguishes normal from abnormal behavior and that takes into account the new results in set-membership diagnostic approaches. Considering these results, which are dedicated to uncertain systems, enlightens the diagnostic reasoning with new aspects. To validate the theoretical aspects, the paper ends by the presentation of an application of diagnostic reasoning and set-membership coherency tests on a laboratory plant: a water tanks system.

**Key words.** Fault diagnosis reasoning, fault detection and isolation, model-based diagnosis, set-membership coherency tests.

## I. INTRODUCTION

Fault diagnosis reasoning became an important research theme since the eighties when the artificial intelligence community began to focus on quality problems in the microelectronics industries. The wave of approaches that was born during this period was followed by the two major contributions: [de Kleer et al, 1987] and [Reiter, 1987]. Most of the works has been gathered in [Hamscher et al, 1992]. Today, these approaches remain very interesting because firstly the obtained diagnosis lies on system components and secondly, it distinguishes diagnosis based on normal behavior from diagnosis based on abnormal one, also called abductive approach. However, artificial intelligence approaches are not very relevant to dynamical systems. In the nineties, the automatic control community had proposed an alternative approach known as fault detection and isolation (FDI) approach. Papers like [Frank, 1996], [Isermann, 1997] or [Patton et al, 1991] represent states of the art of FDI approach. This approach is dedicated to dynamical systems, but it does not consider the system components and both normal and abnormal behavior reasoning are mixed that take away from a rigorous diagnostic reasoning. Recently, [Cordier et al, 2000] made a link between both approaches.

This paper focuses on diagnostic reasoning for uncertain dynamical systems. It proposes a diagnostic reasoning that distinguishes normal from abnormal behavior and that takes into account the new results in set-membership diagnostic approaches [Ploix et al, 2000], [Adrot et al, 1999] and [Armenghol et al, 2000]. Considering these results which are dedicated to uncertain systems, enlightens the diagnostic reasoning with new aspects. To validate the theoretical aspects, the paper ends by the presentation of an application

of diagnostic reasoning and set-membership coherency tests on a laboratory plant: a water tanks system.

To understand how diagnostic reasoning may handle set-membership coherency tests for dynamical systems, the characteristics of these tests have to be examined. A major aspect of set-membership approaches is that they are characterized by insensibility to false alarms: if a false alarm occurs, the coherency test results can be expressed as "The behavior complies or not with the behavioral reference", would remain correct. The presence of a false alarm does not change the test results but it means that the modeled behavior does not represent all the possible behaviors in a given state. The false alarms rely on the first diagnostic step: the modeling, which is uncoupled from coherency tests step (and which is itself uncoupled from diagnostic reasoning).

Another characteristic of set-membership approaches is that the information available about physical phenomena (or physical variables) is not necessarily perfect: it may be imprecise. Thus, in the following, a physical variable will be distinguished from the information about it.  $\tilde{x}$  will denote an information on a phenomenon whereas  $x$  will denote the corresponding phenomenon. In contrast with the phenomenon  $x$  which is unknown,  $\tilde{x}$  will be called: "known variable". A known variable may be considered as exact (for example, in the case of a perfect sensor) or as imprecise.

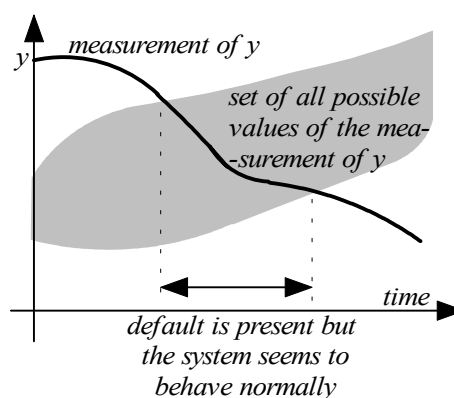


Figure 1-Strict sense of incoherence

In the framework of set-membership diagnostic approaches, the exoneration assumption (namely, a component behaving normally is in a normal state) cannot be admitted for a diagnosis at any time. Figure 1 reveals indeed that the observed incoherences ( $\tilde{y}$  outside the domain delimited by the dotted line) must not be cancelled by some transient coherent results. It appears that if an incoherence occurs at

only one moment, it is more meaningful than any other coherent result for the same test: an incoherence always reveals an abnormal state that will go on as long as the physical system state could not be changed (it requires often a human intervention). However, a second level that formulates the exoneration assumption can complete a diagnosis obtained without the exoneration assumption. But, this level only checks the system behavior and not the system state.

## II. A NEW FAULT DIAGNOSIS REASONING

### II.1. Diagnostic reasoning problem statement

A normal or an abnormal behavior can generally be modeled by elementary analytical relations (EAR) coming from physics. A validity domain and a support can be associated to any of these relations. The validity domain is defined when the coherency test makes sense whereas the support depicts the components, which are responsible for the modeled behavior. In the case of abnormal behavior, it is useful to keep the references of the normal behavior EAR, which are substituted, to optimize the triggering of abnormal behavior tests.

Any diagnostic procedure relies on coherency tests between behavioral knowledge (models) and information on actual state (observations). By definition, these coherency tests can only be achieved using analytical redundancy relations (ARR) which are defined as analytical relations without unknown variables (i.e. without physical variables standing for phenomena). [Staroswiecki et al, 1989] proposes an ARR generation technique based on the simultaneous elimination of a set of variables in a subset of analytical relations. However, this technique cannot take into account the causality (in the sense of calculability [Iwasaki et al, 1994]) in EAR. Consequently, we propose to eliminate variables one by one until an ARR is obtained. The starting point is the same in both approaches: the incidence matrix of the structure. Nevertheless, due to the bringing-in of set-membership aspect, we suggest to distinguish 3 categories of variables: the unknown physical variable  $y$ , the perfectly known variable and the imprecisely known variable, for instance,  $\tilde{y} = (1 + \theta)y$ , where  $\theta$  is an uncertain parameter.

In the case of a causal analytical relation (for example, an ordinary differential equation), any input variable will be denote by the letter 'I' in the element of the incidence matrix at the intersection corresponding to the analytical relation and to the variable and a letter 'O' will denote an output variable. In the case of a non-causal relation, the letter 'P' will be used to denote that a variable is present in an analytical relation. It is assumed, in the proposed technique, that the causal relations only have one output variable. Otherwise, a MIMO model should be decomposed into several MISO models.

### II.2. ARR generation

The set of minimal ARR covers all the parts of a physical system that may be monitored and it allows generating all the other possible ARR, which will be qualified by opposition as composed ARR. A minimal ARR can be computed by the exclusive elimination of the unknown variables between the EAR thanks to the elimination rules given in appendix. The set of minimal ARR is the set of all the distinct ARR that can be found according to this technique. Contrary to the approach proposed in [Staroswiecki et al, 1989], the unknown variables are eliminated one by one in envisaging all the elimination possibilities.

The second step consists in the generation of the composed ARR; it only makes sense for the normal behavior level. The composed ARR are generated from the minimal ARR. They come from the elimination of known variables between ARR according to the procedure previously mentioned. However, the elimination of the known variables cannot be proceeded in the same way depending on the known variables are perfectly known or imprecisely. Considering the example in figure 2, where each plain line represent an EAR, each dashed line an ARR and the circles stand for physical variables and octagons for known variables. The minimal ARR are the relations  $R_1$  and  $R_2$  whereas  $R_3$  is a composed ARR.

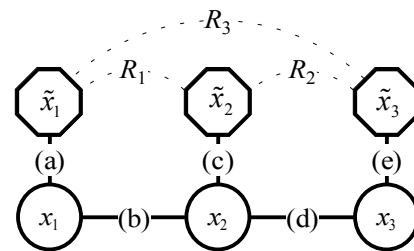


Figure 2 – Schematic chart for the ARR representation

If the elementary analytical relation (c) is certain (i.e. if  $\tilde{x}_2$  represents perfectly the phenomenon  $x_2$ ) and if a fault occurs, for example, on the support of the EAR (a), then, without any exoneration assumption, the coherency test associated to  $R_3$  does not bring any more information than the results obtained from the 2 minimal ARR. Let imagine that the tests based on  $R_1$  and  $R_3$  reveal incoherences and that the test based on  $R_2$  leads to coherency. The tests based on minimal ARR indicate that there is a conflict between the assumption of normal state of components of the supports of EAR (a), (b) and (c). The information resulting from  $R_3$  tends to add to the suspect components, those that belong to the support of (d) and (e). However, at the same time, the coherency tests based on the EAR  $R_2$ , which covers the EAR {c,d,e}, does not reveals any incoherence. Now, because the relation (c) is certain, this test is at least as precise than the one based on  $R_3$  which total the uncertainties on (a), (b), (d) and (e) whereas  $R_2$  totals only uncertainties on (d) and (e). In this way, the ARR  $R_3$  does not bring any more information than  $R_1$  and  $R_2$ ; it follows that (d) and (e) relations appearing in the conflict coming from  $R_3$  must not be taken into account. On the other hand, if relation (c) is imprecise, the

previous reasoning becomes false: an incoherence may be detected on  $R_3$  and not at the level of the minimal ARR.

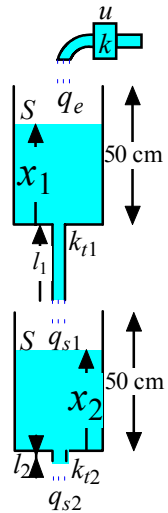
Consequently, without any exoneration assumption, the composed ARR coming only from the elimination of perfectly known variables does not bring more information than the minimal ARR. That is why the imprecisely known variables have to be distinguished. The ARR coming exclusively from the elimination of perfectly known variables will be named "minor composed ARR" (by contrast, the other composed ARR will be called "major composed ARR"); they will be considered only at the second level when the exoneration assumption is done.

### II.3. Coherence tests

According to the previous development, the conflicts (see [de Kleer et al, 1987]) have to be firstly generated from the minimal and major composed ARR when the conditions on the corresponding validity domains are satisfied. Any EAR that appears only one time in a conflict will remain suspect as long as an intervention on the physical system occurs.

In addition, all the coherency results should be stored and, if they remain coherent along time, they could be taken into account for a second level diagnosis with an exoneration assumption to complete the first level of diagnosis. This will be discussed in the section IV.

## III. APPLICATION OF THE FAULT DIAGNOSIS REASONING



An application concerning 2 water tanks is presented here as illustration. Thanks to a digital to analog converter, a Kammer gate controls an input water flow, which pours into an upper tank. At the same time the upper tank is filling, the water it contains can flow through an underneath restriction into a lower tank. The water contained in the tank can also flow by an underneath restriction to be drain away. The water height in upper tank (denoted by  $x_1$ ) and in lower tank (denoted by  $x_2$ ) is measured thanks to precise condenser sensors. A digital controller controls this physical system. An interesting point for fault diagnosis is that the information comes from the process with a sample time of 10s (the controller does use all the information).

Table 1 gathers the elementary models coming from physics with their validity domain and their support in term of components. EAR (a), (c) and (e) contain uncertainties: the uncertain parameters  $k$ ,  $k_{r1}$  and  $k_{r2}$  are supposed to be time varying: they respectively belong to  $[(1-\rho_k)k, (1+\rho_k)k]$ ,  $[(1-\rho_{k_{r1}})k_{r1}, (1+\rho_{k_{r1}})k_{r1}]$  and  $[(1-\rho_{k_{r2}})k_{r2}, (1+\rho_{k_{r2}})k_{r2}]$ . Variables  $v_k$ ,  $v_{k_{r1}}$  and  $v_{k_{r2}}$  are uncertain normalized bounded variables i.e. we only know that their values, which may vary at any time, belong to the interval  $[-1,1]$ .

Ref.	EAR	Validity domain	Support
(a)	$q_e = (1 + \rho_k v_k) k u^3 + \varepsilon_1$	low frequency	Kammer gate
(b)	$S \frac{dx_1}{dt} = q_e - q_{s1}$	$0 < x_1 < 50 \text{cm}$	upper tank
(c)	$q_{s1} = (1 + \rho_{k_{r1}} v_{k_{r1}}) \cdot k_{r1} (x_1 + l_1) + \varepsilon_u$	-	upper tank restriction
(d)	$S \frac{dx_2}{dt} = q_{s1} - q_{s2}$	$0 < x_2 < 50 \text{cm}$	lower tank
(e)	$q_{s2} = (1 + \rho_{k_{r2}} v_{k_{r2}}) \cdot k_{r2} (x_2 + l_2) + \varepsilon_l$	-	lower tank restriction
(f)	$\tilde{u} = u$	low frequency	DA converter
(g)	$\tilde{x}_1 = x_1$	low frequency	$x_1$ sensor
(h)	$\tilde{x}_2 = x_2$	low frequency	$x_2$ sensor

**Table 1** –Elementary analytical relations for the water tanks system

The incidence matrix of the structure (table 2) can be directly drawn from table 1. The left part contains the unknown variables whereas the right one contains the perfectly known variable (there is no imprecisely known variables). The lower part of the table gathers the EAR that are also called observation relations.

		physical phenomena (unknown variables)						perfectly known variables		
		$u$	$q_e$	$q_{s1}$	$q_{s2}$	$x_1$	$x_2$	$\tilde{u}$	$\tilde{x}_1$	$\tilde{x}_2$
Elementary analytical relations	(a)	P	P							
	(b)		I	I		O				
	(c)			P		P				
	(d)			I	I		O			
	(e)				P		P			
	(f)	P						P		
	(g)					P			P	
	(h)						P			P

**Table 2**-Incidence matrix of the water tanks system structure

By eliminating the unknown variables, only two minimal analytical redundancy relation  $R_1$  and  $R_2$  can be found (see table 3). Notice that the columns in the right part of the table correspond to the elementary analytical relation depicted in table 2. A letter 'X' means that the corresponding EAR intervenes in an ARR. This information is useful to generate the conflicts from coherency tests.

Because of the absence of imprecisely known variables, the composed ARR is of course minor.

		known variables			elementary analytical relations							
		$\tilde{u}$	$\tilde{x}_1$	$\tilde{x}_2$	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
min.A RR	$R_1$	I	O		X	X	X			X	X	
	$R_2$		I	O			X	X	X		X	X
comp ARR.	$R_3$	I		O	X	X	X	X	X	X		X

**Table 3-** Obtained analytical redundancy relations

## IV. COHERENCY TESTS SYNTHESIS AND RESULTS

### IV.1. Parameters estimation

The models corresponding to the minimal ARR are first order models. If uncertainties are considered, the discrete model for the upper tank is (for more explanation, see [Ploix et al, 2000b]):

$$\begin{cases} x_{1,k+1} = \left(1 - \rho_{k_{t1}} \frac{k_1 T_e}{S} \nu_{k_{t1}}\right) e^{-\frac{k_1 T_e}{S}} x_{1,k} + \dots \\ \frac{k}{k_{t1}} \left(1 - e^{-\frac{k_1 T_e}{S}}\right) \left(1 + \rho_{k_{t1}} \left(1 - \frac{k_1 T_e}{S \left(1 - e^{-\frac{k_1 T_e}{S}}\right)}\right) \nu_{k_{t1}} + \rho_k \nu_k\right) u_k^3 + \varepsilon_{1,k} \end{cases}$$

A zero-order hold is used because the variable is controlled. The discrete model with uncertainties for the lower tank is:

$$\begin{cases} x_{2,k+1} = \left(1 - \rho_{k_{t2}} \frac{k_2 T_e}{S} \nu_{k_{t2}}\right) e^{-\frac{k_2 T_e}{S}} x_{2,k} + \dots \\ \frac{k_1 S}{k_{t1} T_e} \left[ \frac{1}{k_{t2}^2} \left(1 - \frac{k_2 T_e}{S}\right) e^{-\frac{k_2 T_e}{S}} - 1\right] + \rho_{k_{t2}} \left[ \left(\frac{T_e^2}{S^2} - \frac{2}{k_{t2}}\right) e^{-\frac{k_2 T_e}{S}} + \frac{2}{k_{t2}^2}\right] \nu_{k_{t2}} + \dots \\ \frac{\rho_{k_{t1}}}{k_{t2}^2} \left(1 - \frac{k_2 T_e}{S}\right) e^{-\frac{k_2 T_e}{S}} - 1 \nu_{k_{t1}} \right] x_{1,k} + \dots \\ \frac{k_1 S}{k_{t2} T_e} \left[ \frac{1}{k_{t2}} \left(1 + \frac{k_2 T_e}{S} - e^{-\frac{k_2 T_e}{S}}\right) + \rho_{k_{t2}} \left(\frac{2}{k_{t2}} + \frac{T_e}{S}\right) \left(e^{-\frac{k_2 T_e}{S}} - 1\right)\right] \nu_{k_{t2}} + \dots \\ \frac{\rho_{k_{t1}}}{k_{t2}} \left(1 + \frac{k_2 T_e}{S} - e^{-\frac{k_2 T_e}{S}}\right) \nu_{k_{t1}} \right] x_{1,k+1} + \varepsilon_{2,k} \end{cases}$$

Here, a first-order hold is used because the input of the model is continuous.

The variables, which have to be identified, are the central and uncertainty parameters:  $k$ ,  $k_{t1}$ ,  $k_{t2}$ ,  $l_1$ ,  $l_2$ ,  $\rho_k$ ,  $\rho_{kt1}$ ,  $\rho_{kt2}$ ,  $\varepsilon_u$  and  $\varepsilon_l$ . The identification is decomposed into two steps. It is due to the important number of parameters. The first step concerns the identification of central parameters, and the identification of uncertainty parameters is made in the second step. If necessary, this procedure has to be iterated to optimize the solution.

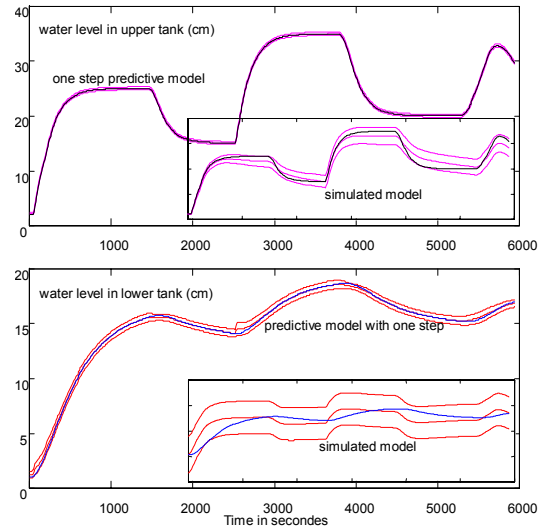
Two data sets of measures that remain in the validity domains are necessary to make a representative model of the possible behaviors at a given state. Measurement filtering is also necessary to get a model valid for low frequencies. Some data sets may lead to trouble in validation stage. The reason

essentially comes from the no representativeness of the data set. The model will be identified one more time with another data set.

Uncertainty parameters are then identified (see also [Ploix et al, 1999]). They will be determined by minimizing the surface, which is delimited by the envelopes:  $J = \int (y_{sup} - y_{inf})(t) dt$ ,  $y_{sup}$  represents the upper envelope and  $y_{inf}$  represents the lower envelope, under the following constraint:

$$\forall t, y_{inf}(t) < \tilde{y}(t) < y_{sup}(t).$$

Two types of identification have been made: identification based on one step ahead predictive model, and identification based on a simulated model. The predictive model uses past values of measures and the input to predict the next value. The simulated model is initialized and builds the output only with the input and past model output. The final models explain all the behavior of the process (figure 3).



**Figure 3-** Coherency tests results in normal behavior

The identified parameter values for the predictive model are the following ones:  $k=1.6064 \cdot 10^{-4} \text{cm}^3 \cdot \text{s}^{-1}$ ,  $k_{t1}=0.100402 \text{cm}^2 \cdot \text{s}^{-1}$ ,  $k_{t2}=0.4942366 \text{cm}^2 \cdot \text{s}^{-1}$ ,  $l_1=70 \text{cm}$ ,  $l_2=3 \text{cm}$ ,  $\rho_k=1.52467\%$ ,  $\rho_{kt1}=62.03190\%$ ,  $\rho_{kt2}=1.05105\%$ ,  $\varepsilon_u=0 \text{cm}^3 \cdot \text{s}^{-1}$  and  $\varepsilon_l=0.105612 \text{cm}^3 \cdot \text{s}^{-1}$ .

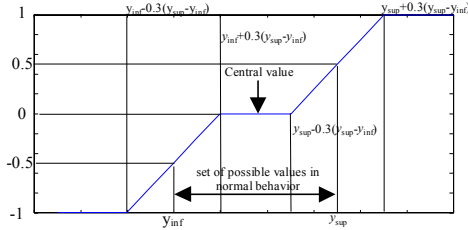
The identified parameter values for the simulation model are the following ones:  $k=1.5 \cdot 10^{-4} \text{cm}^3 \cdot \text{s}^{-1}$ ,  $k_{t1}=7.916575 \cdot 10^{-2} \text{cm}^2 \cdot \text{s}^{-1}$ ,  $k_{t2}=0.4273461 \text{cm}^2 \cdot \text{s}^{-1}$ ,  $l_1=70 \text{cm}$ ,  $l_2=3 \text{cm}$ ,  $\rho_k=9.616391\%$ ,  $\rho_{kt1}=0\%$ ,  $\rho_{kt2}=13.13748\%$ ,  $\varepsilon_u=0.4897364 \text{cm}^3 \cdot \text{s}^{-1}$  and  $\varepsilon_l=1.880504 \text{cm}^3 \cdot \text{s}^{-1}$ .

The predictive model leads to larger uncertainty on  $k_{t1}$  than the simulated model, which leads itself to larger uncertainty on  $k$  and  $k_{t2}$ . The possible sets for output variables are calculated thanks to interval calculation [Ploix et al, 2000b].

### IV.2. Coherency tests achievement

There are 3 coherency tests associated to the obtained ARR. Here, each test consists in examining if a measure goes out of its possible values set in normal behavior, and in computing the distance between the closest border of the possible values

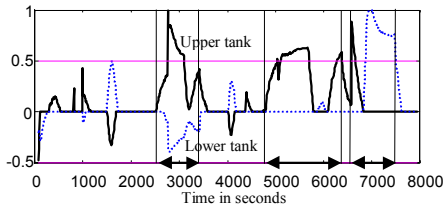
set and the measured value. In order to improve the readability, measures and envelopes coming from possible values sets of the one step predictive models are transformed by the function drawn on figure 4. The possible values set is transformed into a fix interval  $[-0.5, 0.5]$  and the function shape is chosen such as information on the distance to the borders remains in the results.



**Figure 4-** Function that converts a measure into a Boolean information: normal or abnormal behavior

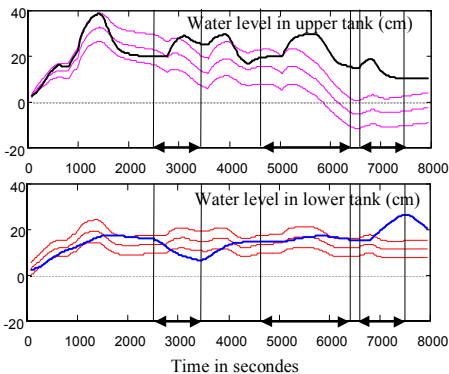
A data set with faults has been stored. Between the time 2490s. and 3400s., an abnormal change of the upper restriction size ( $k_{r1}$ ) occurred. At time 4720s., there was an abnormal additional water flow into the upper tank. It ends at time 6350s. Then, at time 6740s., there was an abnormal additional water flow into the lower tank that ends at time 7460s.

Figures 5 and 6 show that the one step predictive model is closer to the actual behavioral state than the simulation model. It is evident that the simulation model is not so effective because of its largest memory.

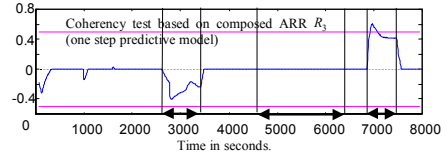


**Figure 5-** Fault detection with the predictive model.

Figure 7 shows that the composed ARR is not so precise that the minimal ones. It is due to the fact that this ARR contains more uncertainties (on  $k$ ,  $k_{r1}$  and  $k_{r2}$ ) than the minimal ARR.



**Figure 6-** Fault detection with the simulation model



**Figure 7-** Fault detection with a composed RRA

### IV.3. Interpretation of coherency tests results

The diagnostic reasoning will be developed using the one step predictive models that are proved to be more efficient (see IV.2). First, the minimal ARR only has to be considered because the composed ARR is minor.

Firstly, incoherence appears on the test based on  $R_1$  in the neighborhood of 3000 second. As all the validity conditions are satisfied, the incoherence means that there is a conflict between the EAR  $\{a,b,c,f,g\}$  (see table 3) and then, **the first level diagnosis is: one or more components of the corresponding supports are faulty** (see table 1): **the Kammer gate, the upper tank, the upper tank restriction, the DA converter and/or the  $x_1$  sensor.**

Secondly, the second test based on  $R_2$  leads to results remaining coherent (even if they seem to become incoherent). It can be inferred that the component associated to the EAR implies in  $R_2$  behave normally and so, their state seems to be normal. Considering tables 3 and 1, **the complementary diagnosis is: the upper tank restriction and the  $x_1$  sensor behave normally (they seems to be non-faulty).** However, the complementary diagnosis tends to clear the true faulty component: the upper tank restriction. It happens because the test based on  $R_2$  is not sufficiently precise to reveal an incoherence. This is a justification of the interest of decomposing diagnosis in 2 distinct levels: a one level reasoning would have cleared the true faulty component whereas here, the first level provides always a true information on the components states and the second level, on the component behavior. Notice that sometimes as for the abnormal additional water flow, the second level may be helpful.  $R_3$  ARR does not bring more information here. The rest of the test results could be studied in the same way.

## V. CONCLUSION

In this paper, the diagnostic reasoning has been adapted to set-membership approaches. The proposed reasoning remains generic and it also applies to other coherency test approaches. The guaranty notion inherent to set-membership tests brings a rigor that is fundamental in diagnosis. Thanks to this rigor, the interest of decomposing fault diagnosis in 2 levels has been shown: one level concerning the components state and an another one, their behavior. The first level is the most meaningful whereas the second one sometimes permits with reservation to distinguish the most probable faulty components among all the suspected ones coming from the first diagnostic level. An analytical redundancy relation generation method has been proposed; it applies simultaneously to causal and to non-causal models. Then, it has been shown that perfectly known variables have to be

considered in a different way than the imprecisely known variables. Finally, the proposed diagnostic reasoning has been applied to a laboratory plant where set-membership tests are achieved.

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## VI. APPENDIX: ELIMINATION RULES

In the following, it is assumed that a variable common to 2 relations can always be eliminated. If it were not the case, the corresponding ARR should be a posteriori cancelled.

The principle is to eliminate a common variable to 2 relations (EAR or ARR), when it is possible, to generate a relation whose support corresponds to the union of the original supports except when a known variable disappears: in this case, the corresponding sensor has to be removed from the resulting support. Moreover, if there are several common variables to 2 relations, they must be separately eliminated by taking care that other common variables effectively remain in the resulting relation when a chosen common variable is eliminated. If need be, the incidence matrix should be a posteriori adjusted.

### *Elimination rule for non causal relations*

When the same variable appears in 2 non-causal relations, it can be eliminated to generate a new relation that contains all the union of all the original relation variables except the one that has been eliminated.

### *Elimination rule for causal relations*

Elimination between 2 causal relations is only possible between an output variable and an input one. This remains true in case of feedback, i.e. if the output variable of one relation can be eliminated with input variable of the other and vice-versa. The output variable of the resulting relation corresponds to the non-eliminated output variable and the input variables to the union of the non eliminated original input variables.

### *Elimination rule between causal and non causal relations*

Input or output variable of a causal relation can be eliminated with the same variable appearing in a non-causal relation to generate a new causal relation.

Several cases have to be considered:

- (1) The eliminated variable appears as an input variable in the causal relation. In this case, except for the eliminated variable, the variables of the non causal relation have to be added to the input variable of the original causal relation to become the input variables of the new causal relation. The output remains the output variable of the original causal relation.
- (2) The eliminated variable appears as an output variable in the causal relation and there are no common variables between the input variable of the causal relation and the variable of the non-causal relation. In this case, the elimination will be possible only if the non-causal relation contains only two variables. One is eliminated and the other one becomes the new output. The inputs

of the new causal relation are the same as the one of the original causal relation.

- (3) The eliminated variable appears as an output variable in the causal relation and some input variables of the causal relation are common with some variables of the non-causal relation. In this case, it exists several resulting MISO relations whose output correspond to one of the input variables common with the non causal relation whereas the other variables of the non causal relation, except for the eliminated variable, will be added to the original input variables to generate the new input variables.

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