

# ASYMPTOTIC OBSERVER FOR A NON-LINEAR DESCRIPTOR SYSTEM

**Stéphane Ploix, Didier Maquin, José Ragot**

*Centre de Recherche en Automatique de Nancy - CNRS UA 821  
Institut National Polytechnique de Lorraine  
2, Avenue de la Forêt de Haye  
54 516 Vandoeuvre les Nancy - France  
email: {sploix, dmaquin, jragot}@ensem-u.nancy-fr*

**Abstract:** This communication deals with the problem of state estimation for a class of non linear singular systems. Sufficient conditions for the existence of such observers are provided and the design of an observer is examined. Assuming that the measurement matrix is full row rank involves many calculation simplifications ; therefore special emphasis in the computational aspect of the observer matrices is provided.

**Keywords:** Non-linear systems, State observer, Descriptor systems, Computational methods.

## 1. INTRODUCTION

State observation of non-linear dynamical systems is becoming a growing topic of investigation in the specialised literature (Tsinias, 1989), (Walcott, 1987). The reconstruction of state variables remains a major problem both in control theory and process diagnosis (Magni, 1991). Researcher attention is being particularly focused on the design of adaptive observers for on-line process state estimation. There is increasing awareness that to ensure robustness in performance requires simpler and stable adaptive observer schemes. Linear systems have received considerable attention leading to several stable adaptive observer systems. Linear observers involving unknown inputs have also been developed and analysed (Chang, 1995), (Gaddouna, 1996). Nevertheless, the design of asymptotically stable observers remains a hard task in the non-linear case, even when the non-linearities are fully known.

This note is organised into two sections. The first describes and justifies the structure of the proposed observer, while the second deals with the computation of matrices involved in this observer.

## 2. NON-LINEAR OBSERVER DEFINITION

Assuming a linear part of non-linear descriptor system may be isolate, any non-linear descriptor systems can be written as:

$$E \frac{dx(t)}{dt} = Ax(t) + Bu(t) + f(x(t), u(t)) \quad (1a)$$

$$y(t) = Cx(t) \quad (1b)$$

where  $f$  is a vector of non linear functions which may represent a known non-linearity and where

$$x(t) \in \mathfrak{X}^n \quad u(t) \in U \subset \mathfrak{R}^p \quad y(t) \in \mathfrak{R}^m$$

$$E \in \mathfrak{R}^{q,n} \quad A \in \mathfrak{R}^{q,n} \quad B \in \mathfrak{R}^{q,p} \quad C \in \mathfrak{R}^{m,n}$$

Notice that range of  $u(t)$  is assume to belong to a ball  $U$  of  $\mathfrak{R}^p$ .

First, it should be noticed that the design of an observer for unknown inputs systems may be solved in the same way because they can be reduced to form (1a) (Gaddouna, 1996).

Assume that:

- the matrix  $C$  is full row rank (2a)

- $\text{rank} \left( \begin{bmatrix} E \\ C \end{bmatrix} \right) = n$  (2b)

- $\text{rank} \left( \begin{bmatrix} sE - A \\ C \end{bmatrix} \right) = n \quad \forall s$  (2c)

- $f$  is  $k$ -lipschitz with respect to  $x$  for any vector  $v$  belonging to a ball  $U$ :  
 $\forall v \in U, \forall (x_1, x_2) \in \mathfrak{X}^n \times \mathfrak{X}^n$ ,  
 $\|f(x_2, v) - f(x_1, v)\| \leq k \|x_2 - x_1\|$  (2d)

The hypothesis (2a) indicates that no redundancy between the measurement devices is considered (this assumption is not very restrictive and can always be satisfied by redefining the measurement equation). The second one (2b) may be interpreted as they are enough measurements to compensate the singularity of system (1a); it will be used further for the design of the observer. Then the third one (2c) is the observability condition. At last, hypothesis (2d) is used for bounding the magnitude of the non-linearity.

Our aim is to find a Luenberger observer which asymptotically estimates the state vector  $x(t)$ . In the following proportional observer is used but the proposed technique may be easily extended to proportional-integral observer.

**Proposition.** Let  $S(\theta)$  be a symmetric positive definite matrix function of a positive scalar  $\theta$ ; then, if the four hypothesis (2) hold, the following system is an observer of system 1:

$$\frac{dz(t)}{dt} = Nz(t) + Ly(t) + Gu(t) + \dots$$
 (3a)

$$\dots Rf(\hat{x}(t), u(t)) - S^{-1}(\theta)C^T(C\hat{x}(t) - y(t))$$

$$\hat{x}(t) = z(t) + Ky(t)$$
 (3b)

with

$$\bullet -\theta < \text{Re}(\lambda_i(N)) : \theta > 0 \quad \forall i \in \{1, \dots, r\}$$
 (4a)

$$\bullet RE + KC = I$$
 (4b)

$$\bullet NRE + LC - RA = 0$$
 (4c)

$$\bullet G - RB = 0$$
 (4d)

where  $N, L, G, R, S$  and  $K$  are matrices of proper dimensions,  $R$  is full column rank,  $\lambda_i(N)$  represent the  $i^{\text{th}}$  distinct eigenvalue of  $N$  and  $\theta$  is a positive parameter.

**Proof.** Proceeding by analogy to the classical observer design approach in the linear case, we seek an observer of the form (3a) and (3b).

Let  $e(t)$  be the state reconstruction error defined by:

$$e(t) = x(t) - \hat{x}(t) \\ = REx(t) - z(t)$$
 (5)

Direct derivation of  $e(t)$  yields:

$$\frac{de(t)}{dt} = (N - S^{-1}(\theta)C^TC)e(t) + (RB - G)u(t) \dots \\ \dots + (RA - NRE - LC)x(t) + R(f - \hat{f})$$

where  $f \equiv f(x(t), u(t))$  and  $\hat{f} \equiv f(\hat{x}(t), u(t))$ .

By virtue of (4c) and (4d), the error dynamic is governed by the following differential equation:

$$\frac{de(t)}{dt} = (N - S^{-1}(\theta)C^TC)e(t) + R(f - \hat{f})$$
 (6)

Let us now examine the stability of  $e(t)$  by considering the Lyapunov function:

$$V(t) = \frac{1}{2} e^T(t)S(\theta)e(t)$$
 (7)

The calculation of the derivative of  $V(t)$  with respect to the time  $t$  gives:

$$\frac{dV(t)}{dt} = \frac{1}{2} e^T(t)(N^TS(\theta) + S(\theta)N - 2C^TC)e(t) \dots \\ \dots + e^T(t)S(\theta)R(f - \hat{f})$$
 (8)

According to the Lyapunov theory, if the linear part of (3a) is stable, a symmetric positive definite matrix  $Q$  exists such as:

$$N^TS(\theta) + S(\theta)N - 2C^TC = -Q$$

Let now  $Q$  satisfying the following equality :

$$Q = 2\theta S(\theta)$$

Then, the Sylvester equation may be written as:

$$(N + \theta I_n)^T S(\theta) + S(\theta)(N + \theta I_n) = 2C^TC$$
 (9)

Then, it is well known that this equation admits a unique solution only if the eigenvalues of  $N + \theta I_n$  belongs to right half plane of the complex map i.e. the parameter  $\theta$  has to be chosen such as (4a) is satisfied.

The solution  $S(\theta)$  is given by :

$$S(\theta) = 2 \int_0^{\infty} \left( e^{-(N + \theta I_n)^T \tau} C^T C e^{-(N + \theta I_n) \tau} \right) d\tau.$$

Notice that  $S(\theta)$  is the observability grammian. It will be also regular and so positive definite if  $(N, C)$  is observable which may be expressed as:

$$\text{rank} \left( \begin{bmatrix} sI_n - N \\ C \end{bmatrix} \right) = n \quad \forall s$$

By using the equations (4a) and (4c), the observability condition may be written:

$$\text{rank} \left( \begin{bmatrix} sI_q - RA + (L + NK)C \\ C \end{bmatrix} \right) = n \quad \forall s$$

It is equivalent to:

$$\text{rank} \left( \begin{bmatrix} sI_q - RA \\ C \end{bmatrix} \right) = n \quad \forall s$$
 (10)

The following decomposition may always be achieved:

$$\begin{bmatrix} sI_q - RA \\ C \end{bmatrix} = \begin{bmatrix} R & sK \\ 0 & I_m \end{bmatrix} \begin{bmatrix} sE - A \\ C \end{bmatrix}$$

Because the matrix  $R$  is full column rank, the following property is satisfy :

$$\begin{bmatrix} R & sK \\ 0 & I_m \end{bmatrix} \text{ is full column rank.}$$

The last property yields to:

$$\text{rank} \left( \begin{bmatrix} R & sK \\ 0 & I_m \end{bmatrix} \begin{bmatrix} sE - A \\ C \end{bmatrix} \right) = \text{rank} \left( \begin{bmatrix} sE - A \\ C \end{bmatrix} \right) = n$$

Therefore, the observability of the pair  $(N, C)$  is equivalent to the hypothesis (2c). ✓

Moreover, it may be proven that the eigenvalues of the linear part of the observer have a real part equal to  $-\theta$ .

The Sylvester equation (9) may be reformulated as:

$$\begin{aligned} & (N - S(\theta)^{-1}C^TC + \theta I_n)^T S(\theta) + \dots \\ & \dots + S(\theta)(N - S(\theta)^{-1}C^TC + \theta I_n) = 0 \end{aligned}$$

Let  $\ell_i : i \in \{1, \dots, \rho\}$  be the  $\rho$  distinct eigenvalues of the linear part of the observer  $N - S(\theta)^{-1}C^TC$  which appears in (6) and  $v_i : i \in \{1, \dots, \rho\}$   $\rho$  eigenvectors associated with each distinct eigenvalues.

By multiplying respectively right and left by eigenvector and its conjugate, it yields:

$$\begin{aligned} & v_i^* (N - S(\theta)^{-1}C^TC + \theta I_n)^T S(\theta) v_i + \dots \\ & \dots + v_i^* S(\theta) (N - S(\theta)^{-1}C^TC + \theta I_n) v_i = 0 \\ & \qquad \qquad \qquad \forall i \in \{1, \dots, \rho\} \end{aligned}$$

However, according to the eigenvalues and eigenvectors definition, it is well known that:

$$(N - S(\theta)^{-1}C^TC + \theta I_n) v_i = (\ell_i + \theta) v_i$$

$$\text{then } v_i^* (N - S(\theta)^{-1}C^TC + \theta I_n)^T = (\ell_i^* + \theta) v_i^* .$$

By substitution, it is obtained:

$$2(\text{Re}(\ell_i) + \theta) v_i^* S(\theta) v_i = 0$$

and, since  $S(\theta)$  is positive definite:

$$\text{Re}(\ell_i) = -\theta \quad \forall i \in \{1, \dots, \rho\}$$

Now examine the condition for which observer (2) is stable. Taking (9) into account reduces equation (8) to:

$$\frac{dV(t)}{dt} = -\theta e^T(t) S(\theta) e(t) + e^T(t) S(\theta) R (f - \hat{f}) \quad (11)$$

From (2d), one deduces:

$$e^T(t) S(\theta) R (f - \hat{f}) \leq k \|e(t)\|^2 \sigma_{\max}(S(\theta)R) \quad (12)$$

where  $\sigma_{\max}(\cdot)$  denotes the largest singular value.

Substituting (12) into (11) gives:

$$\frac{dV(t)}{dt} \leq -\theta e^T(t) S(\theta) e(t) + k \|e(t)\|^2 \sigma_{\max}(S(\theta)R) \quad (13)$$

$$\text{with: } e^T(t) S(\theta) e(t) \geq \sigma_{\min}(S(\theta)) \|e(t)\|^2 \quad (14)$$

One deduces from (13):

$$\frac{dV(t)}{dt} \leq (-\theta \sigma_{\min}(S(\theta)) + k \sigma_{\max}(S(\theta)R)) \|e(t)\|^2 \quad (15)$$

According to the Lyapunov point of view, the stability is ensured if:

$$-\theta \sigma_{\min}(S(\theta)) + k \sigma_{\max}(S(\theta)R) < 0 \quad (16)$$

In the case of  $k$  is known, the inequality (16) is satisfied by adjusting the value of  $\theta$ . One deduces from (15):

$$\frac{dV(t)}{V(t)} \leq 2 \left( -\theta + k \frac{\sigma_{\max}(S(\theta)R)}{\sigma_{\min}(S(\theta))} \right) dt \quad (17)$$

thus :

$$V(t) \leq V(0) \exp \left( 2 \left( -\theta + k \frac{\sigma_{\max}(S(\theta)R)}{\sigma_{\min}(S(\theta))} \right) t \right) \quad (18)$$

On a practical point of view, the design of the observer may be done as follows: choose the parameter  $\theta$ , sufficiently large, solve the Lyapunov equation with regard to  $S(\theta)$  and then verify that the constraint (16) holds ; modify eventually the parameter  $\theta$  for satisfying this constraint. Evidently the choice of  $\theta$  is not unique. Considering (18), it is judicious to choose  $\theta$  such that  $\text{Re}(\theta \sigma_{\min}(S(\theta)) - k \sigma_{\max}(S(\theta)R))$  is greater as possible in order to ensure a rapid decreasing of  $V(t)$ .

**Remark:** A second problem may be dealt with. It corresponds to the situation where it is of interest to determine what would be the maximal admissible value of  $k$ :

$$k \sigma_{\max}(S(\theta)R) < \theta \sigma_{\min}(S(\theta)) . \quad (19)$$

### 3. MATRICES COMPUTATION

After having proposed the structure of the observer and the conditions of its existence, we now propose an algorithm to solve the equations defining the matrices of this observer.

It has been demonstrated in the proof of the **proposition** that, if the hypothesis (2c) holds and if the parameter  $\theta$  is chosen such as (4a) is satisfied, the Sylvester equation (9) admits an unique solution  $S(\theta)$ .

A solution of the system (4) may be obtained by using the decomposition, which can always be achieved by transformation of system (1a):

$$C = [I_m \ 0] \quad (20)$$

This decomposition implies the following ones:

$$E = \begin{bmatrix} \xrightarrow{m} & \xleftarrow{n-m} \\ E_1 & E_2 \end{bmatrix} \begin{matrix} \updownarrow_q \\ \updownarrow_q \end{matrix} \quad A = \begin{bmatrix} \xrightarrow{m} & \xleftarrow{n-m} \\ A_1 & A_2 \end{bmatrix} \begin{matrix} \updownarrow_q \\ \updownarrow_q \end{matrix}$$

$$N = \begin{bmatrix} \xrightarrow{m} & \xleftarrow{n-m} \\ N_{11} & N_{12} \\ \xrightarrow{m} & \xleftarrow{n-m} \\ N_{21} & N_{22} \end{bmatrix} \begin{matrix} \updownarrow_m \\ \updownarrow_{n-m} \\ \updownarrow_m \\ \updownarrow_{n-m} \end{matrix} \quad R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \begin{matrix} \updownarrow_m \\ \updownarrow_{n-m} \end{matrix}$$

$$I_n = \begin{bmatrix} \xrightarrow{m} & \xleftarrow{n-m} \\ J_1 & J_2 \end{bmatrix} \begin{matrix} \updownarrow_n \\ \updownarrow_n \end{matrix} : J_1 = \begin{bmatrix} I_m \\ 0 \end{bmatrix} \text{ and } J_2 = \begin{bmatrix} 0 \\ I_{n-m} \end{bmatrix}$$

### 3.1 Computation of matrix R

According to (2b), one may write:

$$\text{rank} \left( \begin{bmatrix} E_1 & E_2 \\ I_m & 0 \end{bmatrix} \right) = n,$$

it follows that  $\text{rank}(E_2) = n - m$  i.e.

$$E_2 \text{ is full column rank.} \quad (21)$$

Thanks to decompositions, equation (4b) may be re-written as:

$$R[E_1 \ E_2] + K[I_m \ 0] = [J_1 \ J_2]$$

which is equivalent to:

$$\begin{cases} RE_2 = J_2 \\ K = J_1 - RE_1 \end{cases} \quad (22a)$$

$$\begin{cases} RE_2 = J_2 \\ K = J_1 - RE_1 \end{cases} \quad (22b)$$

As  $E_2$  is full column rank (21), the solutions of system (22a) are:

$$R = \chi(I_q - E_2 E_2^+) + J_2 E_2^+ \text{ with } E_2^+ = (E_2^T E_2)^{-1} E_2^T$$

where  $\chi$  may be arbitrarily chosen.

Decomposing  $\chi$  as:

$$\chi = \begin{bmatrix} \xrightarrow{q} \\ \chi_1 \\ \chi_2 \end{bmatrix} \begin{matrix} \updownarrow_m \\ \updownarrow_{n-m} \end{matrix},$$

the previous relation may be written:

$$\begin{cases} R_1 = \chi_1(I_q - E_2 E_2^+) \\ R_2 = \chi_2(I_q - E_2 E_2^+) + E_2^+ \end{cases} \quad (23a)$$

$$\begin{cases} R_1 = \chi_1(I_q - E_2 E_2^+) \\ R_2 = \chi_2(I_q - E_2 E_2^+) + E_2^+ \end{cases} \quad (23b)$$

The matrix R has to be full column rank (see (4)).

From (23), the matrix R may be written as:

$$R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \chi_1 & 0 \\ \chi_2 & I_{n-m} \end{bmatrix} \begin{bmatrix} I_q & -E_2 \\ 0 & I_{n-m} \end{bmatrix} \begin{bmatrix} I_q \\ E_2^+ \end{bmatrix}$$

It is easy to choose  $\chi_1$  such that  $\begin{bmatrix} \chi_1 & 0 \\ \chi_2 & I_{n-m} \end{bmatrix}$  is full

column rank ; under this condition:

$$\text{rank}(R) = \text{rank} \left( \begin{bmatrix} I_q \\ E_2^+ \end{bmatrix} \right) = q$$

and R is therefore a full column rank matrix.

### 3.2 Computation of matrix N

By decomposing the matrices which intervene in (4c), it yields:

$$NR[E_1 \ E_2] + L[I_m \ 0] = R[A_1 \ A_2]$$

i.e.

$$\begin{cases} NRE_2 = RA_2 \\ L = RA_1 - NRE_1 \end{cases} \quad (24a)$$

$$\begin{cases} NRE_2 = RA_2 \\ L = RA_1 - NRE_1 \end{cases} \quad (24b)$$

By taking into account (22a), equation (24a) is equal to:

$$N \begin{bmatrix} 0 \\ I_{n-m} \end{bmatrix} = RA_2 = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} A_2 \quad (25)$$

and, by using the partitioned form of N, the following relations are equivalent to (24a):

$$\begin{cases} N_{12} = R_1 A_2 \\ N_{22} = R_2 A_2 \end{cases} \quad (26)$$

As the matrix L may be freely chosen, the equality (24b) is not a constraint. Then, the matrices  $N_{11}$  and  $N_{21}$  may be arbitrarily chosen.

At last, substituting (23) in (26), the matrix N is expressed by:

$$N = \begin{bmatrix} N_{11} & \chi_1(I_q - E_2 E_2^+) A_2 \\ N_{21} & \chi_2(I_q - E_2 E_2^+) A_2 + E_2^+ A_2 \end{bmatrix} \quad (27)$$

The matrices which may be freely assigned can be gathered in a single matrix  $N_0$ :

$$N_0 = \begin{bmatrix} N_{11} & \chi_1 \\ N_{21} & \chi_2 \end{bmatrix} \quad (28)$$

Then, the matrix N may be parametrized as:

$$N = U + N_0 V \quad (29a)$$

where  $N_0$  is an arbitrary matrix and where:

$$U = \begin{bmatrix} 0 & 0 \\ 0 & E_2^+ A_2 \end{bmatrix} \quad V = \begin{bmatrix} I_m & 0 \\ 0 & (I_q - E_2 E_2^+) A_2 \end{bmatrix} \quad (29b)$$

The eigenvalues of N may be freely assigned if the following observability criterion is satisfy:

$$\text{rank} \left( \begin{bmatrix} sI_n - U \\ V \end{bmatrix} \right) = n \quad \forall s$$

By taking into account the expressions of U and V, one must have:

$$\text{rank} \left( \begin{bmatrix} sI_m & 0 \\ 0 & sI_{n-m} - E_2^+ A_2 \\ I_m & 0 \\ 0 & (I_q - E_2 E_2^+) A_2 \end{bmatrix} \right) = n \quad \forall s$$

i.e.

$$\text{rank} \left( \begin{bmatrix} sI_{n-m} - E_2^+ A_2 \\ (I_q - E_2 E_2^+) A_2 \end{bmatrix} \right) = n - m \quad \forall s \quad (30)$$

The previous condition may be re-written as:

$$\begin{aligned} \text{rank} \left( \begin{bmatrix} sI_{n-m} - E_2^+ A_2 \\ (I_q - E_2 E_2^+) A_2 \end{bmatrix} \right) &= \dots \\ \dots &= \text{rank} \left( \begin{bmatrix} I_{n-m} & 0 \\ E_2 & -I_q \end{bmatrix} \begin{bmatrix} sI_{n-m} - E_2^+ A_2 \\ (I_q - E_2 E_2^+) A_2 \end{bmatrix} \right) = n - m \end{aligned}$$

from which one deduces:

$$\begin{aligned} \text{rank} \left( \begin{bmatrix} sI_{n-m} - E_2^+ A_2 \\ sE_2 - A_2 \end{bmatrix} \right) &= \text{rank} \left( \begin{bmatrix} E_2^+ \\ I_q \end{bmatrix} (sE_2 - A_2) \right) = \dots \\ \dots &= \text{rank}(sE_2 - A_2) = n - m \end{aligned}$$

Taking into account (2c) and the partition of E and C, one have:

$$\text{rank} \left( \begin{bmatrix} sE - A \\ C \end{bmatrix} \right) = \text{rank} \left( \begin{bmatrix} sE_1 - A_1 & sE_2 - A_2 \\ I_m & 0 \end{bmatrix} \right) = n$$

which involves:

$$\text{rank}(sE_2 - A_2) = n - m \quad (31)$$

Thus, because of the condition (2c) the eigenvalues of N can always be freely assigned by adjusting  $N_0$  of (29a) and then the condition (4a) can always be satisfied.

Summarising, the design of this state observer has to be done according the following sequence. First, verify that the hypotheses (2) are satisfied and that the basis is such that C is written as (20). Then, choose parameter  $\theta = \theta_{\max}$  such as the dynamic of the linear part is convenient. Second, assign the eigenvalues of N (29) such as (4a) is satisfied. Then compute R thanks to (23) and, at last, compute L using (24b), K and G are obtained from (22b) and (4d). Solve (9) for different parameter  $\theta \leq \theta_{\max}$  until (14) becomes true.

#### 4. CONCLUSION

A simple method has been presented for designing state-observers of non-linear singular systems. Assuming that the non-linear function is globally k-Lipschitz, it is permitted to set the dynamics of the linear part while ensuring the global exponential stability. Necessary and sufficient conditions for existence and stability of the proposed observer have been established.

#### REFERENCES

- Boutayeb M., M. Darouach, HJ. Rafaralahy, G. Krzakala (1994). Asymptotic observer for a class of non linear singular system. *Proceeding of the ACC*, Baltimore, pp1440-1551.
- Chang S.K., P.L. Hsu (1995). A novel design for the unknown input fault detection observer. *Control Theory and Advanced Technology*, **10** (4), pp1029-1051.

- Gaddouna B., S. Giuliani, J. Ragot (1996). Design of observers for descriptor systems affected by unknown inputs. *International Journal of Systems Science*, **27**(5), pp465-471.
- Kaprielan S., J. Turi (1986). An observer for a non linear descriptor system, *Proc. of 25th IEEE Conference on Decision and Control*, Tucson, pp975-976.
- Magni J.F. and P. Mouyon (1991). A generalised approach to observers for fault diagnosis. *Proc. of the 30th IEEE Conference on Decision and Control*, Brighton, **3**, pp2236-2241.
- Misawa E.A. and J.K. Hedrick (1989). Non-linear observer - A state of the art survey. *Transactions of the ASME*, **111**, pp344-352.
- Söffker D., T.J. Yu, P.C. Müller (1995). State estimation of dynamical systems with non-linearities by using proportional-integral observer. *International Journal of Systems Sciences*, **26** (9), pp1571-1582.
- Walcott B.L. and S.H. Zak (1986). Observation of dynamical systems in the presence of bounded non-linearities / uncertainties. *Proc. of 25th IEEE Conference on Decision and Control*, Athens, pp961-966.
- Walcott B.L. and S.H. Zak (1987). State observation of non-linear uncertain dynamical systems. *IEEE Trans. Automatic Control*, **32**, pp166-170.
- Xia X.H. and W.B. Gao (1988). On exponential observer for non-linear systems. *Systems and Control Letters*, **11**, pp319-325.
- J. Tsiniias (1989). Observer design for non-linear systems. *Systems and Control Letters*, **13**, pp135-142.