

Sensor Placement and Diagnosability

Abed Alrahim Yassine, Stéphane Ploix, Jean-Marie Flaus

G-SCOP, BP 46, Saint Martin d'Herès 38402, France
abed-alahim.yassine@lag.ensieg.inpg.fr,
stephane.ploix@inpg.fr, jean-marie.Flaus@inpg.fr

Abstract. Maintenance and diagnosis of complex systems are common activities in the industrial world. Technological advances have led to a continuously increasing complexity of industrial systems. This complexity, which is due to an increasing number of components reduces in turn the reliability of plants. Therefore, fault diagnosis is becoming a growing field of interest. But fault diagnosis relies on sensors: efficient fault diagnosis procedures require a relevant sensor placement. This paper presents a method for sensor placement based on diagnosability criteria. It makes it possible to determine a sensor placement, which satisfies specifications diagnosability specifications. This method does not require the design of any analytical redundancy relation.

Keywords: fault diagnosis, diagnosability, sensor placement

1. INTRODUCTION

Sensor placement decisions depend on expected objectives. For instance, in control theory, the sensor placement is used to provide sufficient information for the control of systems. Criteria deal with observability and controllability of the variables. (Madron and Veverka, 1992) has proposed a sensor placement method which deals with linear system. This method makes use of the Gauss-Jordan elimination to find a minimum set of variables to be measured. This ensures the observability of variables while simultaneously minimizing the cost of sensors. In this theory, the observable variables include the measurable variables plus the unmeasured but deductible variables.

However, in fault diagnosis, the goal of sensor placement should be to satisfy detectability and diagnosability properties. Detectability is the possibility of detecting a fault on a component and diagnosability is the possibility of identifying a fault on a component without this creating ambiguity with any other single fault.

(Travé-Massuyès *et al.*, 2001) has proposed a method based on consecutive additions of sensors, which takes into account diagnosability criteria. The principle of this method is to analyze the physical model of a system from a structural point of view. This structural approach is based on Analytical Redundancy Relations (ARR) (Cassar and Staroswiecki, 1997), which can be obtained from combinations of model constraints using bi-partied graph (Blanke *et al.*, 2003) or elimination rules (Ploix *et al.*, 2005), and on the correspond-

ing signature table (Patton and Chen, 1991). In a signature table, rows and columns represent respectively, the set of analytical redundancy relations and the set of considered faults. However, this method requires an a priori design of all the ARR for a given set of sensors. Unfortunately, up to now, no method has been able to guarantee to provide all the possible ARR.

This paper presents a method which improves the possibility of detecting and localizing faults in systems of which only the structure is known. Thanks to this method, the sensor placement satisfying diagnosability objectives becomes possible without designing ARR a priori. It is an important feature since it is no longer necessary to design all the possible ARR assuming all the variables are measured.

2. Problem formulation

A system Σ can be described by a tuple $(V_\Sigma, K_\Sigma, C_\Sigma, \Phi_\Sigma)$. V_Σ is the set of variables that models observable phenomena related to Σ . The behaviour is represented by constraints $K_\Sigma = \{\dots, k_i, \dots\}$ that established relationships between variables of V_Σ . It can be summarized by a structural matrix \mathcal{M}_Σ , which is an incidence matrix representing the application $\mathcal{M}_\Sigma : V_\Sigma \rightarrow K_\Sigma$. $C_\Sigma = \{\dots, c_j, \dots\}$ is a set of independent components constituting Σ . Each constraint $k_i \in K_\Sigma$ models a component $c_j \in C_\Sigma$ satisfying $c_j = \Phi_\Sigma(k_i)$ where $\Phi_\Sigma : K_\Sigma \rightarrow C_\Sigma$ ¹.

Let us introduce the concept of testable subsystem (TSS) and its relationship with the concept of ARR.

Definition 1 *A set of constraints is testable if all the constraints can be merged into at least one global constraint that no longer contains variables. The values appearing in the global constraint may correspond either to model parameters or to observations coming from measurements or from controlled variables.* \square

Definition 2 *A testable set of constraints is minimal if it is not possible to keep testability when removing a constraint.* \square

The global constraint that can be deduced from a TSS is named ARR.

Let $R_\Sigma = \{\dots, r_k, \dots\}$ be the set of all the testable subsystems (TSS) that can be deduced from K_Σ according to (Staroswiecki and Declerck, 1989; Blanke *et al.*, 2003; Ploix *et al.*, 2005). Because of the one-to-one relationships between constraints and components, notions of detectability and discriminability can be extended to constraints.

Let R be a set of TSS coming from $(V_\Sigma, K_\Sigma, C_\Sigma, \Phi_\Sigma)$. A constraint $k \in K_\Sigma$ is non detectable in R if $\forall r_i \in R, k \notin r_i$. By extension, the constraints $K \subset K_\Sigma$ are non detectable in R if $\forall k_i \in K, k_i$ is non detectable in R .

Let R be a set of TSS coming from $(V_\Sigma, K_\Sigma, C_\Sigma, \Phi_\Sigma)$. Two constraints $(k_1, k_2) \in K_\Sigma$ are non discriminable in R if: $\forall r_i \in R, k_1 \in r_i \Leftrightarrow k_2 \in r_i$. By extension, the constraints of a set $K \in K_\Sigma$ are non discriminable in R if: $\forall (k_i, k_j) \in K_\Sigma^2, k_i$ and k_j are non discriminable

¹A component may also be modeled by several constraints but, for the sake of simplicity, it has not been considered in this paper.

in R with $k_i \neq k_j$.

Obviously, non detectability implies non discriminability.

Let R be a set of TSS coming from $(V_\Sigma, K_\Sigma, C_\Sigma, \Phi_\Sigma)$. A constraint $k \in K_\Sigma$ is diagnosable in R if: it is detectable and $\forall k_j \in (K_\Sigma \setminus k)$, (k, k_j) are discriminable in R . By extension, the constraints $K \in K_\Sigma$ are dignosable in R if: $\forall k_i \in K$, k_i are diagnosable in R .

In order to formulate the sensor placement problem, the notion of terminal constraint has to be introduced.

Definition 3 A terminal constraint k_i is a constraint that satisfies: $\text{card}(V(k_i)) = 1$ where $V(k_i)$ is the set of variables appearing in the constraint k_i . A terminal constraint usually models a sensor or an actuator. It is thus a major concept in sensor placement. \square

In fault diagnosis, sensor placement has to satisfy specifications dealing with detectability and diagnosability. Therefore, the components C_Σ may be decomposed into several sets:

- the set of components C_{diag} / constraints K_{diag} that has to be diagnosable
- the set of subsets of components $C_{nondis} = \{\dots, C_i, \dots\}$ / constraints $\mathbb{K}_{nondis} = \{\dots, K_i, \dots\}$ that has to be non discriminable but detectable for each set C_i or K_i .
- the set of components C_{nondet} / constraints K_{nondet} that has to be non detectable

Specifications C_{diag} , C_{nondis} and C_{nondet} of sensor placement problems are meaningful if the two following properties are satisfied:

1. Sets in specifications must not to overlap one each other to make sense: constraint sets have to satisfy: $C_{nondet} \cap C_{diag} = \phi$, $\forall C_i \in C_{nondis}, C_i \cap C_{nondet} = \phi$, $\forall C_i \in C_{nondis}, C_i \cap C_{diag} = \phi$ and $\forall (C_i, C_j) \in \mathbb{C}_{nondis}^2, C_i \cap C_j = \phi$ if $C_i \neq C_j$ (no overlapping property).
2. The union of all the components appearing in C_{diag} , C_{nondis} and C_{nondet} has to correspond to C_Σ : $C_\Sigma = C_{diag} \cup C_{nondet} \cup \bigcup_{C_i \in C_{nondis}} C_i$ (completeness property).

If these properties are satisfied the specifications are qualified as consistent in C_Σ . Replacing components by corresponding constraints leads to the same properties for specifications K_{diag} , \mathbb{K}_{nondis} and K_{nondet} to be consistent in K_Σ .

Satisfying the specifications requires information delivered by sensors. Let Σ' represent the system Σ with the additional sensors. Σ' can be described by a tuple $(K_{\Sigma'}, C_{\Sigma'})$ where $C_{\Sigma'}$ represents the components of system Σ plus the additional sensors and $K_{\Sigma'}$ represents the constraints of system Σ plus the additional terminal constraints which model the sensors.

The sensor placement problem consists in determining the additional terminal constraints in $K_{\Sigma'}$ that lead to the satisfaction of the specification K_{diag} , \mathbb{K}_{nondis} and K_{nondet} . Because of the relations between constraints and components, the results can be extended to components.

3. Preliminary concepts

Before deducing diagnosability properties of constraint sets, some concepts have to be introduced.

3.1. Some characteristics of constraint sets

A path p in a subset of constraints $K \subset K_\Sigma$ is an alternate list $(k_1, v_1, k_2, v_2, \dots, k_{n-1}, v_{n-1}, k_n)$, satisfying:

1. $\forall k_i \in p, k_i \in K$
2. $k_i \neq k_j$ if $i \neq j$
3. $v_i \in V(k_i) \cap V(k_{i+1})$
4. $v_i \neq v_j$ if $i \neq j$

A path p is qualified as complete in K if it involves all the constraints of K . The variables involved in a complete path p are denoted $V(p)$. If there is a complete path between a subset of constraints K , K is a constraint set linked by $V(p)$.

A subset of constraints $K \subset K_\Sigma$ is *linked by path* in K_Σ if there is at least one complete path p linking K such that $V(p) \cap V(K_\Sigma \setminus K) = \emptyset$. The shape of a structural matrix corresponding to constraints that are linked by path in K_Σ is drawn in figure 1.

A set of constraints $K \subset K_\Sigma$ isolated by a path p in K_Σ if it is linked by path p and if there is at least one variable $v \in (V_\Sigma \setminus V(p))$ such that $v \in V(K)$ and $\{v\} \cap V(K_\Sigma \setminus K) = \emptyset$. The variable set $V(K) \setminus V(K_\Sigma \setminus K)$ are qualified as *stump* in K_Σ . The set of all stump variables of a set K into K_Σ is denoted $V_S(K_\Sigma, K)$. The shape of a structural matrix corresponding to constraints isolated by path in K_Σ is drawn in figure 2.

One constraint $\{k\} \in K_\Sigma$ is named *stump constraint* in K_Σ if there is at least one variable $v \in V_\Sigma$ such that $v \in V(k)$ and $\{v\} \cap V(K_\Sigma \setminus \{k\}) = \emptyset$. The variables set $V(k) \setminus V(K_\Sigma \setminus \{k\})$ are also qualified as stump in K_Σ with respect to $\{k\}$. The set of all these variables is denoted $V_S(K_\Sigma, \{k\})$. The shape of a structural matrix corresponding to a stump constraint in K_Σ is drawn in figure 2.

3.2. Value propagation as a theoretical tool

According to the definition, a TSS can be modelled by a subset of paths that starts and ends by terminal constraints. An ARR corresponding to a TSS can be seen in different ways. The most common approach is to consider an ARR as a constraint. Another way is to think an ARR as a complete value propagation w.r.t. variables i.e. a propagation that lead to an information about the consistency of a set of constraints, including terminal constraints that

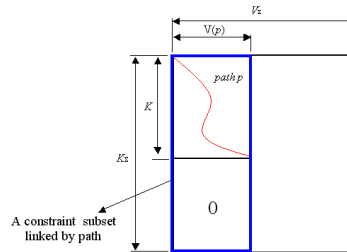


Fig. 1. Structural matrix of a constraint set, which is linked by path

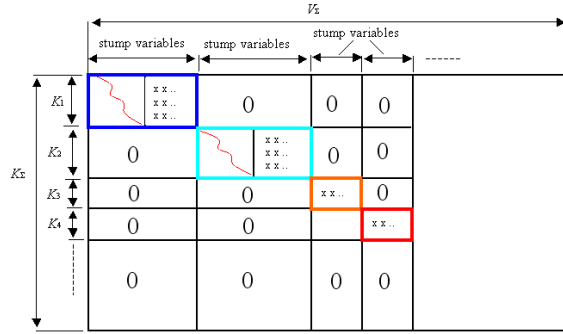


Fig. 2. Structural matrix of non detectable constraints

contains known data. This approach has been adopted as a theoretical tool to develop proofs.

4. Constraint set and diagnosability properties

This section aims at setting up a direct link from problem formulation to detectability, discriminability and diagnosability properties.

Lem 1 Let K be a set of constraints. If additional terminal constraints dealing with each variable in $V(K)$ are added, then all the constraints K are diagnosable. \square

pf 1 Consider one constraint $k \in K$. Because it exists additional terminal constraints dealing with each variable in $V(k)$, it exists an TSS containing k and additional terminal constraints but not other constraints in $K \setminus \{k\}$. Therefore, each constraint $k \in K$ is necessary diagnosable. \square

Lem 2 A necessary and sufficient condition for a subset of constraints $K \subset K_\Sigma$ to be non detectable is that there exists a partition $P(K)$ of K such as each element $K_i \in P(K)$:

- is isolated by a path p_i when there are several constraints in K_i
- corresponds to a stump constraint when there is only one constraint in K_i

and there are no additional terminal constraints containing the stump variables in K_Σ with respect to all the different K_i . \square

Figure 2 points out the shape of a structural matrix of non detectable constraints.

pf 2 Two situations may occur depending on the constraint sets $K_i \in P(K)$. If K_i is composed of only one constraint that contains a variable v , which is absent from other constraints, it is necessarily non detectable, providing that there is no additional terminal constraint dealing with this variable. Indeed, the value of v cannot be propagated and, therefore, no TSS can contain K_i . Consider now the case where K_i is composed of several

constraints isolated by a path p_i . It means that the constraints K_i come always together and that there exists stump variables, which do not belong to the path p_i . Let's name these variables V_i^+ . Because variables of V_i^+ are absent of other constraints $K_\Sigma \setminus K_i$, their value cannot be propagated and, providing that there are no additional terminal constraints dealing with these variables, constraints K_i are necessarily non detectable.

Conversely, a constraint is non detectable if it does not appear in any TSS. It means that at least one variable cannot be propagated. Except for the obvious case where the constraint is stump in K_Σ , it exists necessarily a propagation within a subset of constraints $K \in K_\Sigma; \{k\} \in K$ where the variables $V(K)$ are stump in K_Σ with respect to K . Therefore, K is necessarily isolated by path. \square

Consider for example a system modelled by the structural matrix drawn in table 1.

Table 1. Structural matrix

	v_1	v_2	v_3	v_4	v_5	v_6
k_1	1	0	0	1	0	0
k_2	0	1	1	0	1	0
k_3	0	1	1	0	1	0
k_4	0	0	0	1	0	1
k_5	0	0	0	1	1	1

Assume that the set $K = \{k_1, k_2, k_3\}$ is requested to be non detectable. In this example, it exists a partition $P(K) = \{\{k_1\}, \{k_2, k_3\}\}$ such as each element $K_i \in P(K)$ verifies lemma 2. If there are no additional terminal constraints containing v_1, v_2 and v_3 , the subset K is non detectable.

Lem 3 A sufficient condition for a subset of constraints $K \subset K_\Sigma$ to be non discriminable is that the constraints of K are linked by a path p and that there are no additional terminal constraints containing a variable of $V(p)$. \square

pf 3 Firstly, consider the situation where if K is isolated by p in K_Σ . Then, according to lemma 2, the constraint set K is non detectable and consequently, non discriminable. Secondly, consider situations where K is only linked by path. Two cases may arise: there is only one path p or there are alternative paths p_i satisfying $\forall p_i, V(p) = V(p_i)$. Whatever the case is, the values of these variables can only be deduced by the propagation defined by the path p or possibly by alternative paths p_i , providing that there are no additional terminal constraints dealing with the variables $V(p)$. Therefore the constraints of K will always come together in TSS. i.e $\forall (k_i, k_j) \in K^2, k_i \in TSS \Leftrightarrow k_j \in TSS$, then k_i and k_j are non discriminable in TSS with $k_i \neq k_j$. Consequently the constraints of K cannot be discriminated. \square

Consider for example the system drawn in table 1.

Assume that $K = \{k_2, k_3\}$ is a constraint subset that should be non discriminable. Because the constraints k_2, k_3 are linked by the path $p = (k_2, v_2, k_3)$, lemma 3 is satisfied. Therefore, k_1 and k_2 are non discriminable provided that no additional terminal constraints

contains a variable of $V(p)$.

The following theorem gathers lemmas 1, 2 and 3.

thm 1 *Let (V_Σ, K_Σ) be a model and K_{nondet} , \mathbb{K}_{nondis} and K_{diag} be the specifications of a sensor placement problem consistent in K_Σ . The specifications are fulfilled if:*

1. *it exists a partition $P(K_{nondet})$ such that $\forall K \in P(K_{nondet})$, K is either isolated by a path if there are several constraints in K or, if $K = \{k\}$, k is a stump constraint. The set of all the stump variables is denoted $V_S = \bigcup_{K \in P(K_{nondet})} V_S(K_\Sigma, K)$*
2. *each constraint set $K \in \mathbb{K}_{nondis}$ is linked by a path in $K_\Sigma \setminus K_{nondet}$. The set of all the variables belonging to a path is denoted $V_P = \bigcup_{K \in \mathbb{K}_{nondis}} V(p)$ where p is a linking path in K*
3. *there are no additional terminal constraints containing stump variables coming from 1 and variables of the paths used in 2 i.e. $V_S \cup V_P$*
4. *additional terminal constraints are added for all the remaining variable $V_\Sigma \setminus (V_S \cup V_P)$*
□

pf 4 *The proof directly follows up 2, 3 and 1. The first item of theorem 1 correspond to the direct application of lemma 2. Because there are non detectable, the constraints in K_{nondet} have to be removed from K_Σ before applying lemma 3. According to lemma 1, if additional terminal constraints are added on remaining variables $V_\Sigma \setminus (V_S \cup V_P)$, the constraint in K_{diag} are diagnosable.* □

Satisfying this theorem guarantees that the specifications are satisfied and conversely, if conditions cannot be satisfied, it means that the specifications cannot be satisfied. However, because lemma 1 provides only a necessary condition for diagnosability, the number of additional terminal constraints is not necessary minimal. It has to be checked afterwards.

The sensor placement problem has been studied without considering components. Let's now take components into account. Components of a system may be divided into three sets: the components on which faults need to be isolated, the components on which faults need to be detected but not necessarily localized and the components on which faults need to be non detectable. Because it has been assumed that each component is modeled by only one constraint, the results obtained for constraints can be extended to components using the application $\Phi_\Sigma : K_\Sigma \rightarrow C_\Sigma$.

5. Application to DAMADICS benchmark

Several methods for fault isolation have been benchmarked on a pneumatic servo-motor actuated valve named DAMADICS (Development and Application of Methods for Actuator diagnosis in Industrial Control Systems). (Spanache and Escobet, 2004) has designed a sensor placement method for this problem that optimize the diagnosability level of the system. In this section, the proposed method in this paper, is applied on this benchmark in order to propose a sensor placement that satisfies diagnosability specifications.

The system is defined by the following equations:

$k_1 : X = r_1(P_s, \Delta P)$, $k_2 : F_V = r_2(X, \Delta P)$
 $k_3 : CVI = r_3(SP, PV)$, $k_4 : P_s = r_4(X, CVI, P_z)$
 $k_5 : PV = r_5(X)$

The corresponding structural matrix is given in table 2.

Table 2. structural matrix of DAMADICS

	X	P_s	CVI	PV	F_V	P_z	SP	ΔP
k_1	1	1	0	0	0	0	0	1
k_2	1	0	0	0	1	0	0	1
k_3	0	0	1	1	0	0	1	0
k_4	1	1	1	0	0	1	0	0
k_5	1	0	0	1	0	0	0	0

Let's fix these specifications: $K_{nondet} = \{k_1, k_2\}$, $\mathbb{K}_{nondis} = \{\{k_3, k_4\}\}$ and $K_{diag} = \{k_5\}$.

The set of constraints $K_{nondet} = \{k_1, k_2\}$ is linked by the path $\{k_1, \Delta P, k_2\}$. Because of variable F_V , K_{nondet} is isolated by the path $\{k_1, \Delta P, k_2\}$.

The set of constraints $K = \{k_3, k_4\} \in \mathbb{K}_{nondis}$ is linked by the path $\{k_3, CVI, k_4\}$.

According to theorem 1, no terminal constraints containing a variable from $\{\Delta P, F_V, CVI\}$ have to be added i.e. these variables must not be measured.

In order to satisfied the last item of theorem 1, all the variables of the system except $\{\Delta P, F_V, CVI\}$ have to be measured.

The proposed method in (Ploix *et al.*, 2005) has been used to design all the ARR. It has led to this fault signature matrix:

Table 3. Fault Signature Matrix of DAMADICS

TSS	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
TSS_1	0	0	1	1	1	1	1	0	1	1
TSS_2	0	0	1	1	1	0	1	1	1	1
TSS_3	0	0	0	0	1	1	0	1	0	0
TSS_4	0	0	1	1	0	1	1	1	1	0

According to these results, the constraint set, which cannot be discriminated is: $\{k_3, k_4\}$, the constraints set, which cannot be detected is: $\{k_1, k_2\}$ and the diagnosable constraint set is: $\{k_5\}$. Applying the function $\Phi : K_\Sigma \rightarrow C_\Sigma$, it is obvious that the components, which cannot be discriminated are: $\{c_3, c_4\}$ and the components, which cannot be detected is: $\{c_1, c_2\}$. The diagnosable component is: $\{c_5\}$.

The results presented in this paper demonstrate clearly that our method facilitates sensor placement which satisfy diagnosability criteria without designing ARR a priori

6. Conclusion

A new method of sensor placement has been proposed. It manages specifications dealing with set of constraints that have to be diagnosable, non discriminable or non detectable.

The method applies to any system depicted by constraints, which may only be described by the variables appearing in them. Thanks to this method, sensor placements satisfying diagnosability specifications become possible without designing ARR a priori. It is a very important feature since it is no longer necessary to design all the possible ARR assuming all the variables are measured. An algorithm providing solutions to the sensor placement problem that contain a minimum number of sensors will be provided in the near future.

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