PARITY RELATIONS FOR LINEAR DYNAMIC SYSTEMS WITH MULTIPLICATIVE UNCERTAINTIES

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Abstract: This paper proposes a methodology for the design of parity relations for dynamical systems with multiplicative uncertainties. Instead of canceling uncertainties following the example of the so-called robust approaches, uncertain parity relations take uncertainties into account as bounded variables. The method is based on the analysis of zonotopes representing set of possible behaviors. The proposed parity relations design method applies to any uncertain linear system assuming that it is regularly observable. It requires very little computation time: this approach comes down to compute and check linear inequalities at each sample time.

Keywords: uncertain parity relations, set-membership approach, fault detection

1. INTRODUCTION

This paper focuses on parity relations introduced by Chow and Willsky (Chow and Willsky 1984) and improved by Massoumnia and Van der Velde (Massoumnia and Van Der Velde 1988). These techniques are indeed specific and particularly relevant for fault diagnosis because they lead to very short detection time delay. Since 1995, the interest of researchers for taking into account modeling uncertainties in fault detection is growing up. First works dealing with fault detection algorithms based on set-membership approaches have appeared at the end of the nineties ((Ragot *et al.* 1997, Chang *et al.* 1995)). During 1998 and 1999, interval calculus has been firstly used in fault detection of uncertain systems ((Armengol *et al.* 1999, Ploix 1998)) and, since 2000, lots of works have emerged on this topic. First results on parity relation based on set-membership approaches have been proposed in (Adrot *et al.* 2000). Set-membership approaches for fault detection based on the Hansen algorithm in (Didrit 1997, Hansen 1992), and on the worst-case simulation have been proposed in (Puig *et al.* 2002). (Horak and Allison 1990) proposes a method based on the Pontryagin principle to estimate the enclosures of uncertain systems. However, approaches based on Pontryagin principle, on Hansen algorithm or on the worst-case simulation require lots of computations at each sample time, which are generally incompatible with real-time constraint in complex dynamic systems.

This paper focuses on the design of parity relations for systems modeled by linear models with multiplicative uncertainties.

2. PROBLEM STATEMENT

In order to tackle models containing multiplicative uncertainties with a bounding approach, the concept of membership value set is introduced, given as $\mathcal{M}(\,)$, by analogy with the stochastic variables. If X is a bounded variable, in other words, if it is only known by the space to which it belongs, then this space will be given as $\mathcal{M}(\,)$. The notation x should designate a particular realization of X; nevertheless, for the sake of simplicity, the notation of a realization x will be merged with that of the bounded variable X itself. Henceforth, x will designate, alternately, the bounded variable and one of its realizations, and $\mathcal{M}(x)$ will designate the membership value set of the bounded variable. State space models containing multiplicative uncertain parameters are studied in this paper. It should be noted that by uncertain it is meant that uncertain parameters correspond to time varying (values are unknown and may vary from time to time) or invariant variables (values are unkown but constant), of which only bounds are known.

The following notation is used to distinguish between two kinds of variables: unknown physical variables and known values such as measured or controlled values, which are topped by a '~' sign. For example, the following relation can represent the behavior of a sensor: $\tilde{y} = (1+\upsilon)y$ where υ represents an uncertain parameter, y the actual value of a physical variable and $\tilde{y}$ the measured value of y. For the sake of simplicity, uncertainties will be represented by normalized independent bounded variables, of which the membership value set is equal to $[-1,1]$. For instance, a parameter υ whose value belongs to $[2,6]$, will be written $\upsilon = 4 + 2\theta$ where θ is a normalized uncertain variable: $\mathcal{M}(\theta) = [-1,1]$.

Because parity relations involve finite time horizons, it is usual to consider that some bounded variables are time-invariant over the considered time-horizon h (it is an interesting feature of parity relations because it reduced the problem complexity). Therefore, an uncertain normalized bounded variable may be written either $\theta_{i,k}$ if it is time-varying or ϑ_i elsewhere, which means that this uncertain variable is considered as an unknown constant over the horizon h.

A general discrete-time linear state space model with multiplicative normalized uncertainties ϑ and θ_k, is given by:

$$\begin{cases} x_{k+1} & = A(\vartheta,\theta_k)x_k + B(\vartheta,\theta_k)u_k \\ y_k & = C(\vartheta,\theta_k)x_k + D(\vartheta,\theta_k)u_k \end{cases} \qquad (1)$$

where $x_k \in \mathbb{R}^n, \tilde{u}_k \in \mathbb{R}^m, \tilde{y}_k \in \mathbb{R}^p$ and

$$\begin{cases} A(\vartheta,\theta_k) & = A_0 + \sum\limits_i A_i\vartheta_i + \sum\limits_j A_j\theta_{j,k} = A_k \\ B(\vartheta,\theta_k) & = B_0 + \sum\limits_i B_i\vartheta_i + \sum\limits_j B_j\theta_{j,k} = B_k \\ C(\vartheta,\theta_k) & = A_0 + \sum\limits_i C_i\vartheta_i + \sum\limits_j C_j\theta_{j,k} = C_k \\ D(\vartheta,\theta_k) & = A_0 + \sum\limits_i D_i\vartheta_i + \sum\limits_j D_j\theta_{j,k} = D_k \end{cases}$$

The next section presents how to check if a set of known values $\{\tilde{u}_k,\ldots,\tilde{u}_{k+h-1},\tilde{y}_{k+1},\ldots,\tilde{y}_{k+h-1}\}$ is consistent with an uncertain model such as (1).

According to (Chow and Willsky 1984, Massoumnia and Van Der Velde 1988), the observability concept is a key issue in the design of parity relations. However, observability is more complex in the uncertain context because it depends of uncertain parameters. Let's define the regular observability.

Definition. An uncertain system defined by (1) is called regularly observable if the dimension of its observable subspace, defined by the uncertain matrix $O_h(\upsilon), \forall h$, is independent of the uncertainties i.e. if $\forall h \in \mathbb{N}$, $\forall \upsilon \in \mathcal{M}(\upsilon)$, $rank(O_h(\upsilon)) = rank(O_h(0))$ where $O_h(\upsilon)$ is defined by:

$$O_h(\upsilon) = \begin{bmatrix} C_k \\ C_{k+1}A_k \\ \vdots \\ C_{k+h-1}A_{k+h-2}\ldots A_k \end{bmatrix};$$

$$\upsilon = \begin{bmatrix} \vartheta \\ \theta_k \\ \vdots \\ \theta_{k+h-1} \end{bmatrix} = \begin{bmatrix} \upsilon_1 \\ \vdots \\ \upsilon_q \end{bmatrix}$$

In the next, system defined by (1) is assumed to be regularly observable. Therefore, the horizon h to consider is given by the deterministic theory:

$$h/hp \geq rank(O_h(0)) \qquad (2)$$

For the sake of simplicity, only full column rank state space matrices are considered (model reductions may be required in order to get a non redundant state space representation) i.e.:

$$\forall \upsilon / \|\upsilon\|_\infty \leq 1, rank(O_h(\upsilon)) = p \qquad (3)$$

Therefore, the most interesting horizon h corresponds to the smallest h satisfying (2) and (3).

Considering model (1) and stacking relationships between known values for each sample time such as in most parity relation design approaches (Chow and Willsky 1984), the following relation arises:

$$\begin{bmatrix} \tilde{y}_k \\ \vdots \\ \tilde{y}_{k+h-1} \end{bmatrix} - \Gamma(\upsilon) \begin{bmatrix} \tilde{u}_k \\ \vdots \\ \tilde{u}_{k+h-1} \end{bmatrix} - O_h(\upsilon)x_k = 0 \quad (4)$$

where

$$\Gamma(v) = \begin{bmatrix} D_k & 0 & \dots & 0 \\ C_{k+1}B_k & D_{k+1} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ C_{k+h-1}A_{k+h-2}\dots A_{k+1}B_k & C_{k+h-1}A_{k+h-2}\dots A_{k+2}B_k & \dots & D_{k+h-1} \end{bmatrix}$$

If conditions (2) and (3) are satisfied, it is possible to calculate a projection matrix $\Omega(v)$ satisfying:

$$\forall v/\|v\|_\infty \leq 1, \Omega(v)O_h(v) = 0 \tag{5}$$

Therefore, equation (4) has been projected with respect to $\Omega(v)$ in order to remove the unknown vector x_k:

$$\Omega(v)\begin{bmatrix} \tilde{y}_k \\ \vdots \\ \tilde{y}_{k+h-1} \end{bmatrix} - \Omega(v)\Gamma(v)\begin{bmatrix} \tilde{u}_k \\ \vdots \\ \tilde{u}_{k+h-1} \end{bmatrix} = 0 \tag{6}$$

Nevertheless, two problems arise: the computation of $\Omega(v)$ and the way of testing a parity relation such as (6). Section 4 shows that uncertain parity relations affine in the uncertainties can easily be used for detection. Next section shows how to compute matrices of uncertain parity relations affine in the uncertainties.

3. CALCULATING UNCERTAIN PARITY RELATIONS

A linearized numeric solution $\Omega_L(v)$ of Ω_v solving equation (5) can be computed as follow.

Let $v_{[\alpha]}$ be a power product of uncertain variables coming from matrix $O_h(v)$ defined by $v_{[\alpha]} = v_1^{\alpha_1}v_2^{\alpha_2}\dots v_q^{\alpha_q}$ with $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_q]^\top \in \mathbb{N}^q$ and $\rho(v_{[\alpha]}) = \sum_i v_i$, the order of the power product.

Because the matrices of the discrete-time state space model (1) are affine with respect to uncertain variables, any observability matrix $O_h(v)$, defined on a horizon h, can be reformulated as:

$$O_h(v) = \sum_{\alpha \in V(v)} O_{[\alpha]}v_{[\alpha]} \tag{7}$$

where $V(v)$ stands for the set of all vectors α appearing in the products $v_{[\alpha]}$ of $O_h(v)$ and $dim(O_{[\alpha]}) = hp \times n$. Each term $O_{[\alpha]}v_{[\alpha]}$ is called monom.

By extension, the order of a matrix $O_h(v)$ is equal to the maximum order of its monoms i.e.

$$\rho(O_h) = \max_{\alpha \in V(v)}(\rho(v_{[\alpha]}))$$

Let $\Omega(v)$ be a solution of (5). It may also be decomposed as:

$$\Omega(v) = \sum_{\beta \in W(v)} \Omega_{[\beta]}v_{[\beta]} \tag{8}$$

Therefore, the product $\Omega(v)O_h(v)$ has to satisfy: $\forall v/\|v\|_\infty \leq 1$,

$$\left(\sum_{\beta \in W(v)} \Omega_{[\beta]}v_{[\beta]}\right)\left(\sum_{\alpha \in V(v)} O_{[\alpha]}v_{[\alpha]}\right) = 0$$

This condition can be reformulated as: $\forall v/\|v\|_\infty \leq 1$,

$$\sum_{\gamma \in (V(v) \oplus W(v))}\left(\sum_{\alpha \in V(v)/\alpha \leq \gamma} \Omega_{[\gamma-\alpha]}O_{[\alpha]}\right)v_{[\gamma]} = 0$$

where $V(v) \oplus W(v) = \{\gamma/\gamma = \alpha + \beta; \alpha \in V(v); \beta \in W(v)\}$.

This constraint has to be satisfied $\forall v/\|v\|_\infty \leq 1$. Therefore, the previous summation yields several conditions to be satisfied [1]:

$$\forall \gamma \in (V(v) \oplus W(v)), \sum_{\alpha \in V(v)/\alpha \leq \gamma} \Omega_{[\gamma-\alpha]}O_{[\alpha]} = 0 \tag{9}$$

Because $V(v)$ is a subset of $\mathbb{N}^q$, the set of all the constraints (9) generated by the solution matrix Ω may be pre-defined. Computing a constraint (9) requires indeed that α belongs to $\{\alpha/\gamma \geq \alpha \geq 0; \forall \gamma \in (V(v) \oplus W(v))\}$. Moreover, when constraints of maximum order r are searched, the problem complexity can still be reduced. The set of admissible vectors γ is reduced:

$$\forall \gamma \in (V(v) \oplus W(v)), \rho(\gamma - \alpha) \leq r$$

Therefore, the set $S_r(\gamma)$ of vectors γ generated by a solution matrix Ω, of which maximum order is r, is given by:

$$S_r(v) = \{\gamma \in \mathbb{N}^q; \exists \alpha \in V(v)/(\alpha \leq \gamma) \wedge (\rho(\gamma - \alpha) \leq r)\} \tag{10}$$

Then, searching $\Omega(v)$ satisfying (5) can be achieved iteratively according to the order of $\Omega(v)$. The order $\rho(\Omega)$ is initially set to $r = 0$. The list of constraints $S_r(v)$ is computed according to (10) and the constraints are deduced from (9). A solution matrix $\Omega(v)$ satisfying all the constraints is searched (for instance, the constraints may be gathered into a large matrix of which kernel is computed, see example in section 5). If the number of solutions is lower than p (3), the order $\rho(\Omega(v))$ is incremented by 1 and a new global solution $\Omega(v)$ is searched for the new order.

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[1] A vector $v \in \mathbb{N}^n$ is lower (resp. greater) or equal than another vector $w \in \mathbb{N}^n$ if $v_i \leq w_i$ (resp. $v_i \geq w_i$), $i \in \{1, \dots, n\}$.

When the solution matrix $\Omega(\upsilon)$ has been found, monoms, of which maximum order is greater than 1, are neglected (see example in section 5) because the membership domains are expected to be zonotopes i.e. depicted by equation affine in the uncertain variables. Let $\Omega_L(\upsilon)$ be the first order approximation of $\Omega(\upsilon)$:

$$\Omega_L(\upsilon) = \Omega_0 + \sum_{i=1}^{q} \Omega_i \upsilon_i \tag{11}$$

Matrices Ω_i are thus deduced from (9) and (10). Let ζ_i be a canonical vector of $\mathbb{N}^q$ i.e. a null vector where the i^{th} element is 1. Then,

$$\begin{cases} \Omega_0 = \Omega_{[0]} & \\ \Omega_i = \Omega_{[\zeta_i]} & \text{if}\, \zeta_i \in W(\upsilon) \\ \Omega_i = 0 & \text{elsewhere} \end{cases}$$

In order to compute uncertain parity relations, the first order approximation $\Gamma_L(\upsilon)$ of $\Gamma(\upsilon)$ can be deduced from (4):

$$\Gamma_L(\upsilon) = \Gamma_0 + \sum_{i=1}^{q} \Gamma_i \upsilon_i \tag{12}$$

The resulting linear parity relations affine with respect to uncertainties can be written as:

$$\begin{aligned} 0 = (\Omega_0 + \sum_i \Omega_i \upsilon_i) \begin{bmatrix} \tilde{y}_k \\ \vdots \\ \tilde{y}_{k+h-1} \end{bmatrix} \dots \\ \dots - (\Lambda_0 + \sum_i \Lambda_i \upsilon_i) \begin{bmatrix} \tilde{u}_k \\ \vdots \\ \tilde{u}_{k+h-1} \end{bmatrix} \end{aligned} \tag{13}$$

where $\Lambda_0 = \Omega_0 \Gamma_0$ and $\Lambda_i = \Omega_0 \Gamma_i + \Omega_i \Gamma_0$.

4. TESTING UNCERTAIN PARITY RELATIONS

Uncertain parity relations (13) affine in the uncertainties can be checked thanks to a method used for static uncertain system presented in (Ploix *et al.* 2000). Relation (13) can be reformulated in factorizing uncertain terms:

$$\begin{aligned} 0 = \Omega_0 \begin{bmatrix} \tilde{y}_k \\ \vdots \\ \tilde{y}_{k+h-1} \end{bmatrix} - \Lambda_0 \begin{bmatrix} \tilde{u}_k \\ \vdots \\ \tilde{u}_{k+h-1} \end{bmatrix} + \dots \\ \sum_i \left(\Omega_i \begin{bmatrix} \tilde{y}_k \\ \vdots \\ \tilde{y}_{k+h-1} \end{bmatrix} + \Lambda_i \begin{bmatrix} \tilde{u}_k \\ \vdots \\ \tilde{u}_{k+h-1} \end{bmatrix} \right) \upsilon_i \end{aligned} \tag{14}$$

The general principle behind this approach is to construct the zonotope defined by the vector field appearing in (14) then to check whether the origin of the coordinate axes belongs to this domain.

It has been shown that, for static systems, checking a system $M_k + N_k \upsilon = 0$ where $M_k \in \mathbb{R}^n$ and $N_k \in \mathbb{R}^{n \times q}$ amounts to check if:

$$\{0\} \in \mathcal{M}(M_k \upsilon + N_k) \tag{15}$$

where $\mathcal{M}(M_k \upsilon + N_k)$ is a zonotope, which looks like an hypervolume delimited by parallel hyperplans. A zonotope can be decomposed into a set $\mathcal{S}$ of strip constraints S_i defined by directional vectors $H_i \in \mathcal{H}$:

$$-\|H_i^\top M_k\|_1 + H_i^\top N_k \leq 0 \leq \|H_i^\top M_k\|_1 + H_i^\top N_k \tag{16}$$

Then, testing (15) amounts to test all the strip constraint (16). The computation of the different H_i of $\mathcal{H}$ is achieved by solving this equation:

$$H_i M_k \Phi_i = 0\,;\, rank(M_k \Phi_i) = n - 1 \tag{17}$$

where the matrix Φ_i gathers $n-1$ canonical vectors of $\mathbb{R}^m$. At most, then, there are C_m^{n-1} strip constraints.

Once all the strip constraints (16) have been computed, (15), and consequently (14), can be easily checked in verifying that all the strip constraints are satisfied. This test requires few computations comparing to set-membership inversion approaches.

5. EXAMPLE

Consider the following example:

$$\begin{cases} x_{k+1} &= \begin{bmatrix} 0.8 + 0.1\theta_k & 1 \\ 0 & 0.2 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tilde{u}_k \\ \tilde{y}_k &= \begin{bmatrix} 1 + 0.1\vartheta & 0 \end{bmatrix} x_k \end{cases} \tag{18}$$

The horizon h satisfying (2) and (3) is equal to 3. Therefore, this model can be reformulated for a time horizon of 3 sample times as (4):

$$\begin{bmatrix} \tilde{y}_k \\ \tilde{y}_{k+1} \\ \tilde{y}_{k+2} - (1 + 0.1\vartheta)\tilde{u}_k \end{bmatrix} - O_3(\upsilon) x_k = 0$$

where $\upsilon = [\theta_k, \theta_{k+1}, \vartheta]^\top$ and $O_3(\upsilon)$ can be decomposed into monoms:

$$O_3(\upsilon) = \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \\ 0.64 & 1 \end{bmatrix} \upsilon_{[0,0,0]} + \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \\ 0.08 & 0 \end{bmatrix} \upsilon_{[1,0,0]} + \\ \ldots \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.64 & 0.1 \end{bmatrix} \upsilon_{[0,1,0]} + \begin{bmatrix} 0.1 & 0 \\ 0.08 & 0.1 \\ 0.064 & 0.1 \end{bmatrix} \upsilon_{[0,0,1]} + \\ \ldots \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.01 & 0 \end{bmatrix} \upsilon_{[1,1,0]} + \begin{bmatrix} 0 & 0 \\ 0.01 & 0 \\ 0.008 & 0 \end{bmatrix} \upsilon_{[1,0,1]} + \\ \ldots \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.008 & 0.01 \end{bmatrix} \upsilon_{[0,1,1]} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.001 & 0 \end{bmatrix} \upsilon_{[1,1,1]}$$

Consequently, the set $V(\upsilon)$, appearing in (7), is equal to:

$$V(\upsilon) = \{[0,0,0], [1,0,0], [0,1,0], [0,0,1], \ldots \\ [1,1,0], [1,0,1], [0,1,1], [1,1,1]\}$$

Firstly, let's try a solution matrix $\Omega = \Omega_{[0,0,0]}$ satisfying $\rho(\Omega) = 0$. The set $S_0(\sigma)$, appearing in (10), is given by (10):

$$S_0(\sigma) = \{[0,0,0], [1,0,0], [0,1,0], [0,0,1], \ldots \\ [1,1,0], [1,0,1], [0,1,1], [1,1,1]\}$$

It leads to 8 constraints given by (9), which can be gathered into a large matrix. Thus, the solution matrix $\Omega_{[0,0,0]}$ has to satisfy:

$$\Omega_{[0,0,0]} [O_{[0,0,0]} O_{[1,0,0]} O_{[0,1,0]} O_{[0,0,1]} \ldots \\ O_{[1,1,0]} O_{[0,0,0]} O_{[1,0,1]} O_{[1,1,1]}] = 0$$

Unfortunately, the only solution for Ω is Ω=0.

Therefore, a matrix Ω satisfying $\rho(\Omega) = r = 1$ is then considered. The set $S_1(\sigma)$ is computed and the following projection matrix is successfully obtained:

$$\Omega(\upsilon) = \Omega_{[0,0,0]} + \Omega_{[1,0,0]}\theta_k + \Omega_{[0,1,0]}\theta_{k+1} + \Omega_{[0,0,1]}\vartheta \\ \text{with} \begin{cases} \Omega_{[0,0,0]} & = [0.16, -1, 1] \\ \Omega_{[1,0,0]} & = [0.02, 0, 0] \\ \Omega_{[0,1,0]} & = [0, -0.1, 0] \\ \Omega_{[0,0,1]} & = 0 \end{cases}$$

This solution is already linear and does not require any linearization. Matrix $\Gamma(\upsilon)$ has then to be computed. It is given by (4). However, it is not necessary to compute the complete matrix because only terms affine in the uncertain variables of the product $\Omega(\upsilon)\Gamma(\upsilon)$ are considered:

$$\Omega(\upsilon)\Gamma(\upsilon) = \Omega_0\Gamma_0 + \sum_i (\Omega_0\Gamma_i + \Omega_i\Gamma_0)\upsilon_i + \circ(\upsilon^2)$$

Only terms of order 0 and 1 are necessary:

$$\Gamma(\upsilon) = \Gamma_0 + \Gamma_1\vartheta \\ with \begin{cases} \Gamma_0 & = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ \Gamma_1 & = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix} \end{cases}$$

According to (13), the resulting parity relation, affine in the uncertainties, is:

$$(0.16 + 0.02\theta_k)\tilde{y}_k - (1 + 0.1\theta_{k+1})\tilde{y}_{k+1} + \\ \ldots \tilde{y}_{k+2} + (1 + 0.1\vartheta)\tilde{u}_{k+1} = 0$$

Figure 1 represents the variables, which have been simulated in this example. A fault affecting the structure of the matrix C has been simulated between times 10 and 40 and between times 60 and 90.

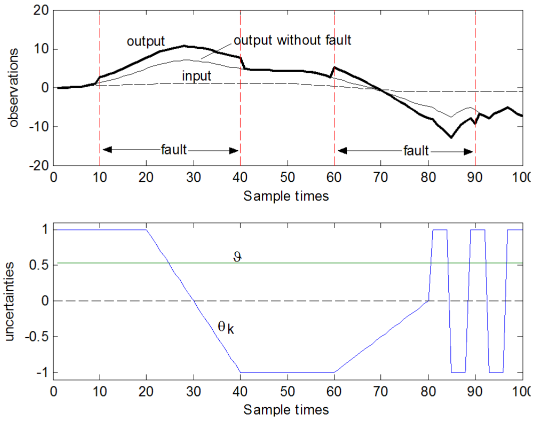

Fig. 1. Simulated variables of the academic example

Figure 2 compares the results of an exact approach (small crosses) based on a set-inversion algorithm (Hansen 1992) and the results of the uncertain parity relation (without any compensation). The abnormal behaviour is detected when fault is present. Faults are still detected out of the faulty time period, for instance at times 41, 91 and 92: it is explained by the time horizon, which is equal to 3 sample times.

Results are identical but set-inversion approach has required about 1 hour with Matlab on a Pentium 4 computer instead of less than 1 second for the uncertain parity relation.

6. CONCLUSION

Uncertain parity relations are powerful tools to handle uncertain dynamic systems where multiplicative uncertainties are predominant. The

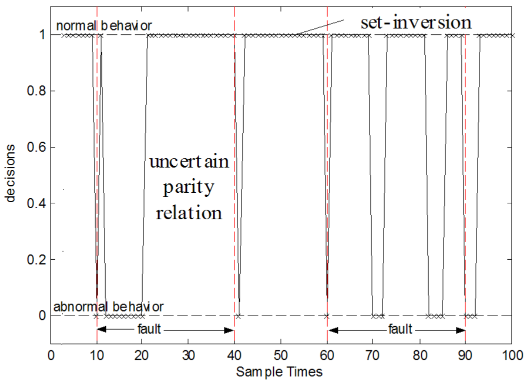

Fig. 2. Detection results of the nonlinear and affin approaches

deterministic approach becomes indeed imprecise because the a posteriori thresholds offsetting the neglected uncertain part of the deterministic model becomes not enough conservative. Uncertain parity relations are interesting alternatives to the set-membership state-estimation approaches that require integration with respect to the time and therefore set the problem of the wrapping effects (Milanese *et al.* 1996), which is often solved in degrading the guaranty property.

The proposed parity relations design method applies to any uncertain linear system assuming that it is regularly observable. It requires very little computation time: this approach comes down to compute and check linear inequalities at each sample time. Even if it requires a linearization, it can be compensated by an additive uncertain variables a posteriori tuned.

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