New results for Sensor Placement with Diagnosability Purpose

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Abstract

Abstract: Maintenance and diagnosis of complex systems are common activities in the industrial world. Technological advances have led to a continuously increasing complexity of industrial systems. This complexity, which is due to an increasing number of components reduces in turn the reliability of plants. Therefore, fault diagnosis is becoming a growing field of interest. But fault diagnosis relies on sensors: efficient fault diagnosis procedures require a relevant sensor placement. This paper presents fundamental results for sensor placement based on diagnosability criteria. These results contribute to the design of sensor placement algorithms, which satisfies specifications composed of several sets: the components for which faults have to be isolable, the components for which faults have to be detectable but not necessarily isolable and the components for which faults have do not need to be detected.
Keywords: fault diagnosis, diagnosability, sensor placement

Introduction

Sensor placement decisions depend on expected objectives. For instance, in control theory, the sensor placement is used to provide sufficient information for the control of systems. Criteria deal with observability and controllability of the variables. (Madron & Veverka 1992) has proposed a sensor placement method which deals with linear system. This method makes use of the Gauss-Jordan elimination to find a minimum set of variables to be measured. This ensures the observability of variables while simultaneously minimizing the cost of sensors. In this theory, the observable variables include the measurable variables plus the unmeasured but deductible variables. Another method for sensor placement has been proposed in (Maquin, Luong, & Ragot 1997). This method aims at guaranteeing the detectability and isolability of sensor failures. The proposed method is based on the concept of redundancy degree in a variable and the structural analysis of the system model. The sensor placement can be solved with a matricial analysis of a cycle matrix or using the technique of mixed linear programming. (Commault, Dion, & Yacoub Agha 2006) has proposed a method of sensor location. In this method, they defined a new set of separators (Irreducible Input Separators), which generates sets of system variables in which additional sensors must be implemented to solve the considered problem.

However, in fault diagnosis, the goal of sensor placement should be to satisfy detectability and diagnosability properties. Detectability is the possibility of detecting a fault on a component and diagnosability is the possibility of identifying a fault on a component without this creating ambiguity with any other fault.

(Travé-Massuyès, Escobet, & Milne 2001) has proposed a method based on consecutive additions of sensors, which takes into account diagnosability criteria. The principle of this method is to analyze the physical model of a system from a structural point of view. This structural approach is based on Analytical Redundancy Relations (ARR) (Cassar & Staroswiecki 1997), which can be obtained from combinations of model constraints using bi-partite graph (Blanke *et al.* 2003) or elimination rules (Ploix, Désinde, & Touaf 2005), and on the corresponding signature table (Patton & Chen 1991). In a signature table, rows and columns represent respectively, the set of analytical redundancy relations and the set of considered faults. However, this method requires an a priori design of all the ARR for a given set of sensors. Unfortunately, up to now, no method has been able to guarantee to provide all the possible ARR.

This paper presents results for the design of sensor placement algorithms. Thanks to these results, the sensor placement satisfying diagnosability objectives becomes possible without designing ARR a priori. It is an important feature since it is no longer necessary to design all the possible ARR assuming all the variables are measured.

Problem formulation

In the following, the set of variables appearing in a constraint k is denoted: $var(k)$ and the set of variables appearing in the set of constraints K: $var(K) = \bigcup_{k \in K} var(k)$.

A system Σ can be described by a tuple (K_Σ, C_Σ). $var(K_\Sigma)$ is the set of variables that models observable phenomena influenced by Σ. The behavior is represented by constraints $K_\Sigma = \{\ldots, k_i, \ldots\}$ that establish relationships between variables of $var(K_\Sigma)$. It can be represented by a structural matrix $\mathcal{M}_\Sigma$, which is an incidence matrix representing the application $\mathcal{M}_\Sigma : var(K_\Sigma) \rightarrow K_\Sigma$. $C_\Sigma = \{\ldots, c_j, \ldots\}$ is a set of independent components constituting Σ. Each constraint in K_Σ models one component and, conversely, a component can be modeled by at most one constraint: $\forall k \in K_\Sigma, comp(k) \in C_\Sigma$[1].

Let us introduce the concept of testable subsystem (TSS) and its relationship with the concept of ARR.

Definition 1 *A set of constraints is testable if all the constraints can be combined into at least one global constraint that no longer contains variables. The values appearing in the global constraint may correspond either to model parameters or to observations coming from measurements or from controlled variables.*

This definition also applies to models containing ordinary differential equations. Indeed, testable state space representations, including state space observers, always have equivalent parity space representations (Staroswiecki, Cocquempot, & Cassar May 5 11 1991).

Definition 2 *A testable set of constraints is minimal if it is not possible to keep testability when removing a constraint.*

A global testable constraint that can be deduced from a TSS is called ARR. Let $R_\Sigma = \{\ldots, r_k, \ldots\}$ be the set of all the testable subsystems that can be deduced from K_Σ according to (Blanke *et al.* 2003; Ploix, Désinde, & Touaf 2005; Staroswiecki & Declerck 1989).

Because of the one-to-one relationships between constraints and components, notions of detectability and discriminability can be extended to constraints.

Let R be a set of TSS coming from (K_Σ, C_Σ)[2].

Definition 3 *A constraint $k \in K_\Sigma$ is detectable (see (Struss* et al. *2002)) in R if $\exists r_i \in R / k \in r_i$. By extension, the constraints $K \subset K_\Sigma$ are detectable in R if $\forall k_i \in K$, k_i is detectable in R.*

Definition 4 *Two constraints $(k_1, k_2) \in K_\Sigma^2$ are discriminable (see (Struss* et al. *2002)) in R if: $\exists r_i \in R / k_1 \in r_i$ and $k_2 \notin r_i$. By extension, the constraints of a set $K \subset K_\Sigma$ are discriminable in R if: $\forall (k_i, k_j) \in K^2$, k_i and k_j are discriminable in R with $k_i \neq k_j$.*

Obviously, non detectability implies non discriminability.

Definition 5 *A constraint $k \in K_\Sigma$ is diagnosable (see (Struss* et al. *2002; Console, Picardi, & Ribando 2000)) in R if: it is detectable and if $\forall k_j \in (K_\Sigma \setminus k)$, (k, k_j) are discriminable in R. By extension, the constraints $K \in K_\Sigma$ are diagnosable in R if: $\forall k_i \in K$, k_i are diagnosable in R.*

In order to formulate the sensor placement problem, the notion of terminal constraint has to be introduced.

Definition 6 *A terminal constraint k is a constraint that satisfies: $card(var(k)) = 1$ where $var(k)$ is the set of variables appearing in the constraint k. A terminal constraint usually models a sensor or an actuator. It is thus a major concept in sensor placement.*

In fault diagnosis, sensor placement has to satisfy specifications dealing with detectability and diagnosability. Because of the one-to-one relation between components and constraints, what is true for components is also true for constraints. Therefore, the components C_Σ and the corresponding constraints K_Σ may be decomposed into several sets:

- the set of components C_{diag} / constraints K_{diag} that has to be diagnosable
- the set of subsets of components $\mathbb{C}_{nondis} = \{\ldots, C_i, \ldots\}$ / constraints $\mathbb{K}_{nondis} = \{\ldots, K_i, \ldots\}$ that have to be non discriminable but detectable for each set C_i or K_i.
- the set of components C_{nondet} / constraints K_{nondet} that has to be non detectable

Specifications C_{diag}, $\mathbb{C}_{nondis}$ and C_{nondet} of sensor placement problems are meaningful if the two following properties are satisfied:

1. Sets in specifications must not to overlap one each other to make sense: constraint sets have to satisfy: $C_{nondet} \cap C_{diag} = \phi$, $\forall C_i \in \mathbb{C}_{nondis}, C_i \cap C_{nondet} = \phi$, $\forall C_i \in \mathbb{C}_{nondis}, C_i \cap C_{diag} = \phi$ and $\forall (C_i, C_j) \in \mathbb{C}_{nondis}^2$, $C_i \cap C_j = \phi$ if $C_i \neq C_j$ (no overlapping property).
2. The union of all the components appearing in C_{diag}, $\mathbb{C}_{nondis}$ and C_{nondet} has to correspond to C_Σ: $C_\Sigma = C_{diag} \cup C_{nondet} \cup \bigcup_{C_i \in \mathbb{C}_{nondis}} C_i$ (completeness property).

If these properties are satisfied the specifications are qualified as consistent in C_Σ . Replacing components by corresponding constraints leads to the same properties for specifications K_{diag}, $\mathbb{K}_{nondis}$ and K_{nondet} to be consistent in K_Σ.

Satisfying the specifications requires information delivered by sensors. Let Σ' represent the system Σ with the additional sensors. Σ' can be described by a tuple $(K_{\Sigma'}, C_{\Sigma'})$ where $C_{\Sigma'}$ represents the components of system Σ plus the additional sensors and $K_{\Sigma'}$ represents the constraints of system Σ plus the additional terminal constraints which model the sensors. The sensor placement problem consists in determining the additional terminal constraints in $K_{\Sigma'}$ that lead to the satisfaction of the specification K_{diag}, $\mathbb{K}_{nondis}$ and K_{nondet}. Because of the relations between constraints and components, the results can be extended to components.

In the next sections, fundamental results are proposed for the design of sensor placement satisfying diagnosability and detectability specifications. Algorithms are not detailed in this paper.

[1]A component may also be modeled by several constraints but, for the sake of simplicity, it has not been considered in this paper.

[2]C_Σ is not used at this stage.

Preliminary concepts

Before deducing diagnosability properties of constraint sets, some concepts have to be introduced.

Value propagation as a theoretical tool

According to the definition, an TSS is a minimum set of constraints K such that there exists a constraint $k \in K$ for which all the variables of $var(k)$ can be instantiated, starting from terminal constraints. An ARR corresponding to a TSS can be seen in different ways. The most common approach is to consider an ARR as a constraint. Another way is to think of an ARR as a complete value propagation (Fron 1994) w.r.t. variables i.e. a propagation that leads to information about the consistency of a set of constraints, including terminal constraints that contain known data. This approach has been adopted as a theoretical tool to develop proofs. Relationship between value propagation and ARR is detailed in this section.

Let k_1 and k_2 be two constraints. The propagation of a variable v between k_1 and k_2 is possible only if $v \in var(k_1) \cap var(k_2)$. The variable v is qualified as propagable between k_1 and k_2. Consider a system, defined by $K_\Sigma = \{k_1, k_2, k_3, k_4, k_5\}$ with $var(k_1) = \{v_1, v_3\}$, $var(k_2) = \{v_1, v_2\}$, $var(k_3) = \{v_2, v_3\}$, $var(k_4) = \{v_2\}$ and $var(k_5) = \{v_3\}$. Terminal constraints k_4 and k_5 model sensors or actuators. Each terminal constraint contains known data. The set of all the tests that can be performed is represented by the propagations drawn in figure 1.

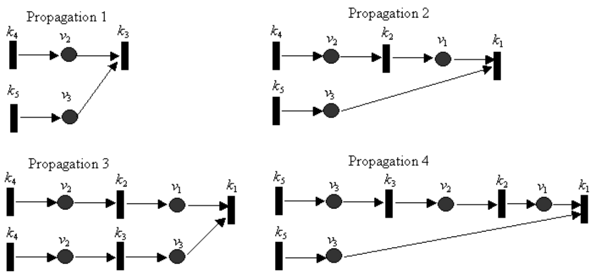

Figure 1: Set of propagation

A propagation starts by a terminal constraint, which means that "a variable is equal to a known value". In this example, propagations start either with k_4 or k_5. Thanks to these constraints, a value can be respectively assigned to v_2 and v_3. Once values have been assigned to these variables, new variables can then be instantiated. Propagation continues until no more assignments are possible because terminal constraints or instantiated variables have been reached. The set of constraints that appears in a propagation, corresponds to a testable subsystem. These constraints can be combined into a unique global constraint named ARR. Depending on the constraints chosen for propagating values, different ARR may be obtained (see figure 1).

In the continuation of this paper, value propagation is implicitly used and appears in the proofs of the different lemmas and theorems.

Some characteristics of constraint sets

The concept of *linked constraints* is introduced because it is important regarding sensor placement. Indeed, discriminability depends on this concept.

As mentioned in (Blanke *et al.* 2003), the constraints of a system Σ may be modeled by a non directed bipartite graph $(K_\Sigma, var(K_\Sigma), E_\Sigma)$ where E_Σ is the set of edges. Each edge $e = (k, v)$ models that $v \in var(k)$.
Let us introduce new definitions useful for sensor placement.

Definition 7 *A set of constraints $K \subset K_\Sigma$ is interconnected by a set of variables $V \subset var(K_\Sigma)$ iff there is a tree $(K, V, E) \subset (K_\Sigma, var(K_\Sigma), E_\Sigma)$ with constraints at extremities (see (Bollobás 1998) for example), which satisfies $card(V) = card(K) - 1$.*

Definition 8 *A set of constraints $K \subset K_\Sigma$ is linked in K_Σ by a set of variables $V \subseteq var(K_\Sigma)$ iff K is interconnected by V and iff the other constraints of K_Σ (i.e. $K_\Sigma \backslash K$) do not contain any variable of V. The variables of V are called linking variables for K. They are denoted: $var_{linking}(K, K_\Sigma)$.*

The shape of a structural matrix dealing with linked constraints is drawn in figure 2.

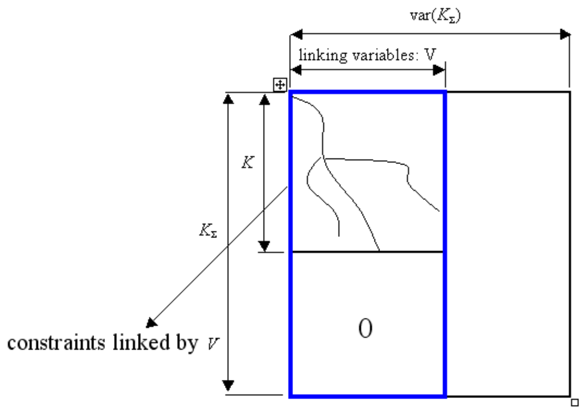

Figure 2: Structural matrix of a constraint set, which is linked by path

The concept of linked constraints is strongly connected with discriminability.

Lemma 1 *A set of constraints $K \subset K_\Sigma$ linked by a set of variables $V \subset V_\Sigma$ is necessarily non discriminable.*

proof 1 *Indeed,*

1. *because variables in V only appear in the constraints in K, the only way of propagating variables is to use the constraints in K and the variables in V,*

2. *because there is a tree $(K, V, E) \subset (K_\Sigma, var(K_\Sigma), E_\Sigma)$ with constraints at extremities, instantiating all the variables in V involves at least the achievement of the propagations defined by the tree.*

Therefore, all the constraints are invariably found together in TSS. In order to improve the clarity of these explanations, let us introduce the notion of stump variables.

Definition 9 *A set of variables $var(K)$ appearing in a set of constraints K but not in the other constraints of K_Σ (i.e. $K_\Sigma \backslash K$) are named stump variables in K_Σ. They are denoted: $var_{stump}(K, K_\Sigma)$.*

For instance, the set of variables V that links a set of constraints K belong to the stump variables $var_{stump}(K, K_\Sigma)$.

A set of constraints cannot be used to generate a TSS if they are linked and if there are additional variables that cannot be propagated. These constraints are qualified as isolated. Detectability depends on this concept.

Definition 10 *A set of several constraints $K \subset K_\Sigma$ is isolated in K_Σ by a set of variables $V \subset var(K_\Sigma)$ if they are linked by V and if there is at least one variable in $var(K) \backslash V$ that does not belong to other constraints of K_Σ (i.e. $K_\Sigma \backslash K$). If the set contains only one constraint, the link condition disappears but the other remains.*

The shape of a structural matrix dealing with isolated constraints is drawn in figure 3.

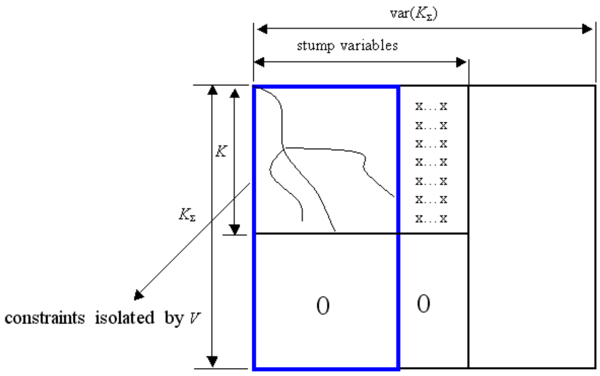

Figure 3: Structural matrix of a constraint set, which is isolated by the set of variables V

The concept of isolated constraints is strongly linked with detectability.

Lemma 2 *A set of constraints $K \subset K_\Sigma$ isolated in K_Σ by V is necessarily non detectable.*

proof 2 *The constraints K isolated in K_Σ by V will always come together in TSS because, by definition, they are linked by V. Because of the fact that, in isolated constraints, there is at least one additional variable in $var(K)$ which does not appear in other constraints (i.e. $K_\Sigma \backslash K$), it is not possible to instantiate this value and, therefore, this set of constraints cannot be involved into a TSS: K is non detectable.*

Constraint set and diagnosability properties

This section aims at setting up a direct link from sets of constraints to detectability and diagnosability properties. Firstly, it is obvious that adding additional constraints connected to all the variables $var(k)$ appearing in a constraint k, ensures the diagnosability of k.

Lemma 3 *Let $k \in K_\Sigma$ be a constraint. If additional terminal constraints dealing with all the variables in $var(k)$ are added, then the constraint k is diagnosable.*

proof 3 *Because there are additional terminal constraints connected to each variable in $V(k)$, a value can be assigned for each variable. Consequently, there is one TSS containing k plus additional terminal constraints connected to variables in $var(k)$. Therefore, the constraint $k \in K$ is necessarily diagnosable because there is one TSS that does not contain other constraints of K_Σ (i.e. $K_\Sigma \backslash \{k\}$).*

Lemma 3 can be directly applied to all the constraints of a constraint set.

corollary 1 *If additional terminal constraints dealing with all the variables $var(K)$ of a constraint set $K \in K_\Sigma$, then each constraint $k \in K$ is diagnosable.*

In lemma 2, a relationship between isolated constraints and the detectability property has been presented. The next lemma generalizes the previous results.

Lemma 4 *A sufficient condition for a subset of constraints $K \subset K_\Sigma$ to be non detectable is that there is a tuple $(K_1, \ldots, K_m)$ of m sets of constraints making up a partition $\mathcal{P}(K)$ of K such that each K_i is isolated in $K_\Sigma \backslash \bigcup_{j<i} K_j$ (K_1 is a limit case: it should be isolated in K_Σ) .*

proof 4 *The case of K_1 has been discussed in lemma 2: because the constraints in K_1 are isolated in K_Σ, they are non detectable and therefore cannot be included in TSS. Then, the remaining candidate constraints for TSS belong to $K_\Sigma \backslash K_1$. Because K_2 is isolated in $K_\Sigma \backslash K_1$, they are non detectable. The reasoning can be extended to any i. Consequently, the constraints in $K = \bigcup_i K_i$ are non detectable.*

Figure 4 indicates the shape of a structural matrix of non detectable constraints.

Consider, for example, a system modeled by the following structural matrix:

	v_1	v_2	v_3	v_4	v_5	v_6
k_1	1	0	0	1	0	0
k_2	0	1	1	0	1	0
k_3	0	1	1	0	1	0
k_4	0	0	0	1	0	1
k_5	0	0	0	1	1	1

Assume that the set $K = \{k_1, k_2, k_3\}$ is required to be non detectable. In this example, there exists a tuple $(\{k_1\}, \{k_2, k_3\})$ such that each element K_i satisfies lemma 4. If there are no additional terminal constraints containing v_1, v_2 and v_3, the subset K is non detectable.

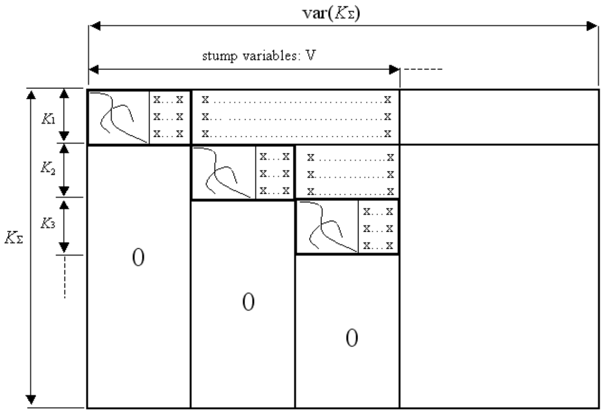

Figure 4: Structural matrix of non detectable constraints

Lemma 5 *A sufficient condition for each set $K_i \subset K$ belonging to a set of m constraint sets $\mathbb{K} = \{K_1, \ldots, K_m\}$ such that $\forall K_i \neq K_j, K_i \cap K_j = \emptyset$, to be non discriminable is that each K_i is linked by a set of variables V_i.*

proof 5 *This lemma is a direct application of lemma 1 to several sets of constraints.*

Consider, for example, a system modeled by the following structural matrix:

	v_1	v_2	v_3	v_4	v_5
k_1	1	0	1	1	1
k_2	1	1	1	1	0
k_3	1	1	1	0	1
k_4	0	1	1	0	0
k_5	0	0	0	1	1

Assume that $K = \{k_1, k_2, k_3, k_4\}$ is a constraint subset that should be non discriminable. Because the constraints k_1, k_2, k_3 and k_4 are linked by $V = \{v_1, v_2, v_3\}$, lemma 5 is satisfied. Therefore, k_1, k_2, k_3 and k_4 are non discriminable provided that no additional terminal constraints contain a variable of V.

The following theorem groups lemmas 3, 4 and 5.

theorem 1 *Let K_Σ be a set of constraints and K_{nondet}, $\mathbb{K}_{nondis}$ and K_{diag} be the specifications of a sensor placement problem consistent in K_Σ. Sufficient conditions for the specifications to be fulfilled are:*

1. *there exists a tuple $(K_1, \ldots, K_p)$ of p sets of constraints making up a partition $\mathcal{P}(K_{nondet})$ of K_{nondet} such that each K_i is isolated in $K_\Sigma \backslash \bigcup_{j<i} K_j$ (K_1 is a limit case: it should be isolated in K_Σ) see figure 4.*
2. *each set K_i belonging to $\mathbb{K}_{nondis} = \{K_1, \ldots, K_m\}$ such that $\forall K_i \neq K_j, K_i \cap K_j = \emptyset$, is linked by a set of variables V_i in considering only the constraints $K_\Sigma \backslash K_{nondet}$*
3. *Additional terminal constraints are added on the variables $V_{candidate} = var(K_\Sigma) \backslash (var_{stump}(K_{nondet}, K_\Sigma) \cup \bigcup_{K_j \in \mathbb{K}_{nondis}} var_{linking}(K_j, K_\Sigma \backslash K_{nondet}))$ (see figure 5).*

proof 6 *The proof relies on the resulting structure of the structural matrix, which directly stems from corollary 1 and lemmas 4 and 5. Note that point 2 could also be stated for the whole set of constraints K_Σ. However, it is not useful to include non detectable constraints, which will not appear in resulting TSS: it would be less conservative.*

Because of lemma 4 and 5, the variables of $var(K_{diag})$ cannot contain variables appearing in the variables involved in (1) and (2) i.e. in $var_{stump}(K_{nondet}, K_\Sigma)$ and in $\bigcup_{K_j \in \mathbb{K}_{nondis}} var_{linking}(K_j, K_\Sigma \backslash K_{nondet})$. Then, $var(K_{diag})$ satisfies: $var(K_{diag}) \subset V_{candidate}$. Because the variables of $V_{candidate}$ can be instantiated with measured values, all the constraints of K_{diag} are diagnosable following corollary 1.

The point that has to be proved is that, in specifications, $\mathbb{K}_{nondis}$ defines non discriminable but detectable sets and not only non discriminable sets as in lemma 5: the detectability of sets in $\mathbb{K}_{nondis}$ has to be proved.

The variables $var(K_i)$ of a constraint set $K_i \in \mathbb{K}_{nondis}$ can be decomposed into two sets: V_i^- and V_i^+ where $V_i^- = var_{linking}(K_i, K_\Sigma \backslash K_{nondet})$ contains the linking variables and V_i^+ contains the remaining variables $V_i^+ = var(K_i) \backslash V_i^-$. Because of lemma 4 and 5, the set V_i^+ cannot contain variables in $var_{stump}(K_{nondet}, K_\Sigma)$ and in $\bigcup_{K_j \in \mathbb{K}_{nondis}; K_j \neq K_i} var_{linking}(K_j, K_\Sigma)$. Therefore, V_i^+ satisfies: $V_i^+ \subset V_{candidate}$

Because of the third point of the theorem, all the variables of $V_{candidate}$ are known: additional terminal constraints are indeed added, there is necessarily a TSS dealing with all the constraints in K_i. It proves that the constraint set K_i is necessarily detectable. Because this result holds for any $K_i \in \mathbb{K}_{nondis}$, it proves the theorem.

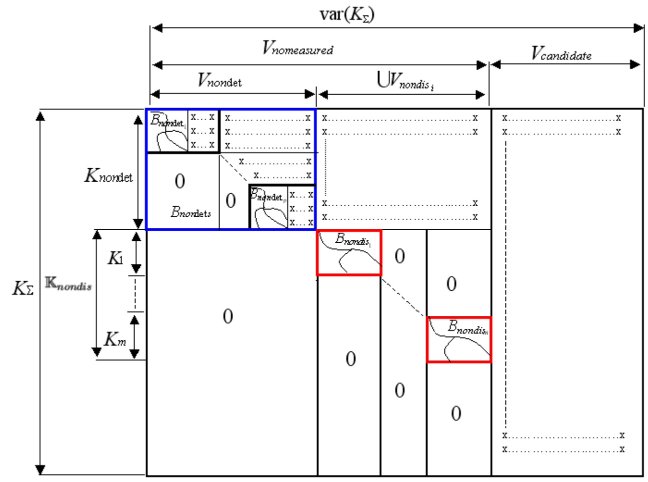

Figure 5: Shape of a structural matrix Satisfying theorem 1

Satisfying theorem 1 guarantees that the specifications are satisfied. However, because the theorem provides only a sufficient condition for diagnosability, the number of additional terminal constraints is not necessarily minimal.

It has to be checked afterwards.

The sensor placement problem has been studied without considering components. Let us now take components into account. Components of a system may be divided into three sets: the components on which faults need to be isolated, the components on which faults need to be detected but not necessarily localized and the components on which faults need to be non detectable. Because it has been assumed that each component is modeled by only one constraint, the results obtained for constraints can be extended to components using the application $\Phi_\Sigma : K_\Sigma \rightarrow C_\Sigma$.

Application to DAMADICS benchmark

Several methods for fault isolation have been benchmarked on a pneumatic servo-motor actuated valve named DAMADICS (Development and Application of Methods for Actuator diagnosis in Industrial Control Systems). (Spanache & Escobet 2004) has designed a sensor placement method for this problem that optimizes the diagnosability level of the system. In this section, the proposed results in this paper are applied on this benchmark in order to compute a sensor placement that satisfies diagnosability specifications.

The system is defined by the following equations:

$$\begin{aligned} k_1 &: \quad x = r_1(ps, \triangle p) \\ k_2 &: \quad fv = r_2(x, \triangle p) \\ k_3 &: \quad cvi = r_3(sp, pv) \\ k_4 &: \quad ps = r_4(x, cvi, pz) \\ k_5 &: \quad pv = r_5(x) \end{aligned}$$

The corresponding structural matrix is given by:

	x	ps	cvi	pv	fv	pz	sp	$\triangle p$
k_1	1	1	0	0	0	0	0	1
k_2	1	0	0	0	1	0	0	1
k_3	0	0	1	1	0	0	1	0
k_4	1	1	1	0	0	1	0	0
k_5	1	0	0	1	0	0	0	0

Let's fix these specifications: $K_{nondet} = \{k_4\}$, $\mathbb{K}_{nondis} = \{\{k_1, k_2\}, \{k_3, k_5\}\}$ and $K_{diag} = \{\phi\}$.

The constraint k_4 is isolated by $\{pz\}$, then the lemma 4 is satisfied.

The set of constraints $K_1 = \{k_1, k_2\} \in \mathbb{K}_{nondis}$ is linked by $\{\Delta p\}$, and the set of constraints $K_2 = \{k_3, k_5\} \in \mathbb{K}_{nondis}$ is linked by $\{pv\}$.

According to theorem 1, no terminal constraints containing a variable from $\{pz, \triangle p, pv\}$ have to be added i.e. these variables must not be measured.

In order to satisfy the last item of theorem 1, all the variables of the system except $\{pz, \triangle p, pv\}$ have to be measured. These measured variables $V_{candidate} = \{x, ps, cvi, fv, sp\}$ represent the candidate variables.

The method proposed in (Ploix, Désinde, & Touaf 2005) has been used to design all the ARR. It has led to this fault signature table:

ARR	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
ARR_1	0	0	1	0	1	1	0	1	0	1
ARR_2	1	1	1	0	1	0	1	1	1	1
ARR_3	1	1	0	0	0	1	1	0	1	0

According to these results, the constraint sets that cannot be discriminated is: $\{k_1, k_2\}$ and $\{k_3, k_5\}$ and the constraint set that cannot be detected is: $\{k_4\}$. Applying the function $\Phi : K_\Sigma \longrightarrow C_\Sigma$, it is obvious that the components that cannot be discriminated are: $\{c_1, c_2\}$ and $\{c_3, c_5\}$ and the component that cannot be detected is: $\{c_4\}$.

Suppose now that the specifications are given by: $\mathbb{K}_{nondis} = \{\{k_3, k_4, k_5\}\}$ and $K_{diag} = \{k_1, k_2\}$.

The set of constraints $\{k_3, k_4, k_5\} \in \mathbb{K}_{nondis}$ is linked by $\{cvi, pv\}$.

According to theorem 1, no terminal constraints containing a variable from $\{cvi, pv\}$ have to be added i.e. these variables must not be measured.

In order to satisfy the last item of theorem 1, all the variables of the system except $\{cvi, pv\}$ have been measured. These measured variables $V_{candidate} = \{x, ps, fv, pz, sp, \triangle p\}$ represent the candidate variable.

The proposed method in (Ploix, Désinde, & Touaf 2005) has been used to produce all the TSS. It leads to the following fault signature matrix:

ARR	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}
ARR_1	0	0	1	1	1	1	1	0	1	1	0
ARR_2	1	1	1	1	1	0	1	1	1	1	0
ARR_3	1	0	1	1	1	0	1	0	1	1	1
ARR_4	1	1	0	0	0	1	1	1	0	0	0
ARR_5	1	1	1	1	1	1	0	1	1	1	0
ARR_6	1	0	0	0	0	1	1	0	0	0	1
ARR_7	0	1	1	1	1	0	1	1	1	1	1
ARR_8	1	1	1	1	1	0	0	1	1	1	1
ARR_9	1	1	0	0	0	0	1	1	0	0	1
ARR_{10}	0	1	0	0	0	1	0	1	0	0	1
ARR_{11}	1	0	1	1	1	1	0	0	1	1	1

According to these results, the constraint set that cannot be discriminated is: $\{k_3, k_4, k_5\}$ and the diagnosable constraint set is: $\{k_1, k_2\}$. Applying the function $\Phi : K_\Sigma \longrightarrow C_\Sigma$, it is obvious that the components that cannot be discriminated are: $\{c_3, c_4, c_5\}$ and the diagnosable component set is: $\{c_1, c_2\}$.

Suppose that the specifications are given by: $\mathbb{K}_{nondis} = \{\{k_3, k_4\}, \{k_2, k_5\}\}$ and $K_{diag} = \{k_1\}$.

The set of constraints $K_1 = \{k_3, k_4\} \in \mathbb{K}_{nondis}$ is linked by $\{cvi\}$, but the set of constraints $K_2 = \{k_2, k_5\} \in \mathbb{K}_{nondis}$ cannot be linked by a set of variables. Consequently, theorem 1 is not satisfied and there is no solution that satisfies these specifications.

The results presented in this paper demonstrate that it is possible to design sensor placements which satisfy diagnosability criteria without designing ARR a priori.

Conclusion

New results for the design of sensor placement algorithms has been proposed. It manages, the specifications dealing with sets of constraints that have to be diagnosable, non discriminable or non detectable. These results apply to any system depicted by constraints, which may only be described by the variables appearing in them. Thanks to these results, sensor placements satisfying diagnosability specifications become possible without designing ARR a priori. It is a very important feature since it is no longer necessary to design all the possible ARR assuming that some variables are measured. An algorithm providing solutions to the sensor placement problem that contains a minimum number of sensors will be provided in the near future.

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